



Hysteretic switching active disturbance rejection control for the servo drives of a skating-training cable robot

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Abstract

Whether a skating-training robot reproduces the prescribed motion depends on whether its servo layer can follow the motor-angle reference amplified by the screw transmission and the cable geometry. Switching active disturbance rejection control improves the large-error response by switching between linear and nonlinear corrections, but the switching frequency and the tracking accuracy lack a quantitative relation, and the hysteresis width is selected without a stated rule. This paper builds an integrated model from cable geometry and screw transmission to the permanent-magnet synchronous motor and gives the torque, speed and voltage margins of each drive; it then proposes a state-preserving hysteretic implementation in which two normalized switching variables act on the observer and feedback corrections separately, with a hysteresis band added. Six controller families are compared at fixed gains with equal tuning effort over 154 cases: the 30 rad step overshoot falls from 52.171% to 0.311%, the maximum error in the load interval falls from 0.101 rad to 0.0447 rad, and the number of mode actions falls from 46 to 18 at a hysteresis width of 0.2. With the switching frequency taken as a design variable, the servo layer can set the switching count to its target while preserving tracking quality, which offers a reusable parameterized method for the servo design of comparable cable robots.

Keywords: active disturbance rejection control; cable robot; permanent-magnet synchronous motor; hysteresis switching

1 Introduction

The propulsion of speed skating relies on the athlete completing a periodic push in a low-drag posture, with coordinated extension of hip, knee and ankle forming the core of the motion [1-3]. Reproducing this motion on land requires a device that constrains the posture while allowing large limb excursions, and a cable-driven mechanism offers a workable option: the motors are fixed to the frame and pull a moving platform through several cables, which removes the drive inertia carried by the limbs and places multiple attachment points around the body [4-9]. The eight-cable robot of this paper follows this layout,



30 in which eight motors drive eight cables through screws with a lead of 4 mm/r and the hip cycle is 3 s at the nominal cadence, corresponding to 20 pushes per minute, so that a 10 min session repeats about 200 push loadings. This transmission decides the role of the servo layer: a change of 1 mm in cable length corresponds to only 0.25 motor revolutions, an amplification of about 1571 rad/m, so the accuracy of motion reproduction ultimately depends on the servo layer.

Control research on cable robots concentrates on tension feasibility. Khosravi and Taghirad [10] embedded internal-force
35 compensation in a robust PID law so that the positive-tension constraint is maintained inside the loop; Babaghasabha et al. [11] extended adaptive robust control to unknown uncertainty bounds, removing the need for a priori bounds; Seyfi and Keymasi Khalaji [12] applied robust control to a lower- and upper-limb rehabilitation cable robot and verified tracking under passive motion. At the mechanism level, Metillon et al. [13] characterized the performance and human-robot interaction quality of collaborative cable robots, and Rousseau et al. [14] proposed human-cable collision detection. These works
40 establish the analysis and control framework for tension feasibility.

Another line of research concerns wearable mechanisms and drive control. For wearable mechanisms, Chen et al. [15], Dollar and Herr [16] and Tucker et al. [17] reviewed the configuration, actuation and control strategies of lower-limb exoskeletons and active orthoses, identifying inertia and alignment near the limb as the main constraints. For switching control, Han [18] proposed estimating and compensating a lumped disturbance with an extended-state observer, Huang and
45 Xue [19] gave its methodology and theoretical analysis, and Gao [20] reduced the gains to be tuned to three by bandwidth parameterization; Qu et al. [21] used linear-nonlinear switching to improve permanent-magnet synchronous motor speed regulation, Hao et al. [22] and Liu et al. [23] applied switching to predictive current control, and Yin et al. [24] and Tang et al. [25] adopted switching in converter and observer structures; Herbst [26], Herbst et al. [27] and Herbst and Hempel [28] discussed bumpless transfer, rate limitation and half-gain observer tuning, Lakomy and Madonski [29] designed thresholds
50 for measurement noise, Zhang et al. [30] and Li et al. [31] treated robust tuning and differentiator efficiency, and Yao and Deng [32], Jiang et al. [33], Wang et al. [34] and Liu et al. [35] applied the related methods to hydraulic servos, delay systems and motor current loops. These works establish feasible paths for cable tension control and for switching control respectively, but the relation between the executable range of a given reference, the switching frequency and the design parameters still lacks a unified quantitative description. Examining reference generation, switching construction and
55 switching frequency within one framework under drive capability constraints is therefore of direct value for the servo layer of cable robots.

To address these issues, this study takes the servo layer of an eight-cable skating-training robot as its object and proposes a state-preserving hysteretic switching active disturbance rejection control implementation. A model from cable geometry and screw transmission to the permanent-magnet synchronous motor is built, and the speed, torque and voltage margins of each
60 drive at a given cadence are given to determine the executable range of the reference. The controller switches the observer and feedback corrections with two normalized switching variables, shares one observer state per drive, and limits reverse switching by a hysteresis band, so that the number of mode actions can be counted from the mode history. On this basis the correspondence between the hysteresis width and the number of mode actions is given, and six controller families are



compared at fixed gains with equal tuning effort over 154 cases. Paired trials and matched-gain ablation separate the roles of the hysteresis band and of the two switching paths. The design places the executable range, the switching construction and the switching frequency within one framework, sets the switching frequency to its target while preserving tracking quality, and thus provides a basis for the parameter design of the servo layer of comparable cable robots.

2 From cable geometry to the servo drive

2.1 Cable geometry and reference generation

The routing of the cables is fixed by eight frame points and eight platform points, shown in Figure 1(a) and Figure 1(b). For a given platform pose the length of cable i is the distance between its two attachment points, which gives:

$$l_i = \|\mathbf{b}_i - \mathbf{p} - \mathbf{R}\mathbf{a}_i\|, \quad i = 1, \dots, 8 \quad (1)$$

where $\mathbf{b}_i$ and $\mathbf{a}_i$ are the attachment positions of cable i on the frame and on the platform, and $\mathbf{p}$ and $\mathbf{R}$ are the platform displacement and orientation. Eight distance constraints cover six pose coordinates, so the mapping from cable length to pose is redundant. Redundancy has two consequences: tension distribution gains a degree of freedom, and the cables no longer share the load equally. The wide spread of per-channel errors in Figure 7 later originates here.

Equation (1) maps the cable length to the platform pose, while the length change must be converted into a motor angle through the screw. With p_h as the lead, the relation between the length change of cable i and the motor angle is:

$$\Delta l_i = l_i - l_i^0, \quad \theta_i^{ref} = \frac{2\pi}{p_h} \Delta l_i \quad (2)$$

where Δl_i is the change in the length of cable i and $p_h = 4 \text{ mm/r}$ is the screw lead. The step is a pure geometric conversion, but its scale factor $2\pi/p_h \approx 1571 \text{ rad/m}$ deserves attention: 1 mm of cable error turns into 0.25 revolutions of motor error. Tracking error is amplified here, and no control law can undo it, which is why this paper concentrates on the servo drive rather than on trajectory planning alone.

Equation (2) links the cable length to the motor angle, but the hip motion itself still has to be described before a reference can be generated. Since the push is periodic, two-term Fourier series express the two hip angles as functions of time:

$$\varphi_j(t) = \varphi_{j0} + \sum_{k=1}^2 [a_{jk} \cos(k\omega t) + b_{jk} \sin(k\omega t)], \quad j = 1, 2 \quad (3)$$

where φ_1 and φ_2 are hip flexion and abduction, φ_{j0} are the offsets, a_{jk} and b_{jk} are the harmonic coefficients, and ω is the push frequency, corresponding to a 3 s period at the nominal cadence. Two terms suffice because the push force is dominated by the fundamental and the second harmonic; higher harmonics displace the platform by less than 1 mm and have been absorbed into the offsets.

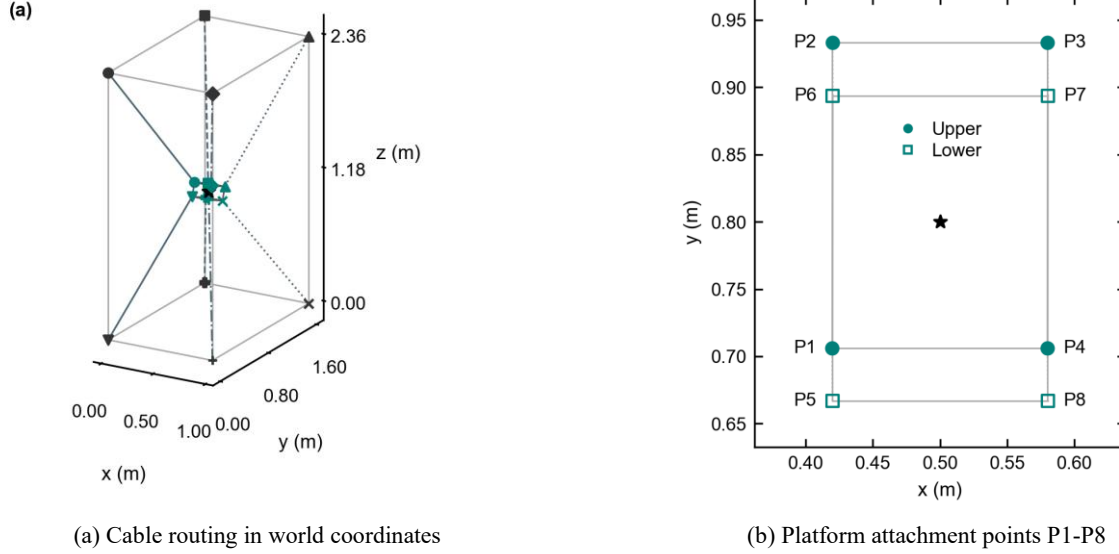


Figure 1. Cable geometry of the training robot.

2.2 Drive chain and motor model

Equation (3) gives the reference trajectory, and substituting the reference pose into Equation (1) gives the cable lengths, but a cable can only pull, so every tension must stay non-negative. The tensions map onto the equivalent force and moment through the structure matrix, that is:

$$\mathbf{J}(\mathbf{q}) = \frac{\partial \mathbf{l}}{\partial \mathbf{q}}, \quad \boldsymbol{\tau} = -\mathbf{J}^T \mathbf{f}, \quad \mathbf{f} \geq \mathbf{0} \quad (4)$$

where $\mathbf{J}$ is the Jacobian of cable length with respect to platform pose, $\mathbf{f}$ is the vector of eight tensions, and $\boldsymbol{\tau}$ is the equivalent force and moment. The condition $\mathbf{f} \geq \mathbf{0}$ separates cable drive from rigid-link drive: if a section of the reference requires a negative tension, that section cannot be realized. All cases evaluated below were selected with this condition satisfied.

Equation (4) maps the tensions to the equivalent force and moment, while the platform is ultimately driven by eight motors through screws, so the electrical model of the drive is required. For a surface-mounted permanent-magnet synchronous motor, the d-axis voltage equation in the dq frame is:

$$u_d = R i_d + L_d \frac{di_d}{dt} - \omega_e L_q i_q \quad (5)$$

where u_d , u_q , i_d and i_q are the dq-axis voltages and currents, R is the stator resistance, L_d and L_q are the dq-axis inductances, and ω_e is the electrical angular speed. The last term is the cross-coupling term and cannot be neglected once the speed rises.



105 Equation (5) relates the d-axis voltage to the d-axis current, and the q-axis equation carries one extra back-electromotive term that grows with speed. To describe the voltage demand at high speed, the q-axis voltage is written as follows, so that Equations (5) and (6) together describe the stator voltage:

$$u_q = R i_q + L_q \frac{di_q}{dt} + \omega_e (L_d i_d + \psi_f) \quad (6)$$

where ψ_f is the permanent-magnet flux linkage. This term lifts the q-axis voltage as the speed increases and is the direct reason why the voltage margin in Figure 4 turns negative beyond 1.5 times the nominal cadence.

110 Equations (5) and (6) give the stator voltage and thus the current dynamics, and the electromagnetic torque combines the q-axis current with the reluctance component. As the quantity that links the electrical and mechanical sides, the torque is:

$$T_e = \frac{3}{2} n_p [\psi_f i_q + (L_d - L_q) i_d i_q] \quad (7)$$

where n_p is the pole-pair number. For a surface-mounted machine $L_d \approx L_q$, the reluctance term can be dropped, and torque becomes proportional to i_q , so a torque reference can be converted into a current reference directly.

Equation (7) gives the electromagnetic torque that acts on the motor shaft, and the mechanical equation connects this torque, the load torque and the speed. It is also the starting point for obtaining the demanded torque from the reference:

$$J_m \frac{d\omega_m}{dt} = T_e - B \omega_m - T_L, \quad \frac{d\theta_m}{dt} = \omega_m \quad (8)$$

120 where J_m is the inertia referred to the motor shaft, B is the viscous damping, T_L is the load torque, and θ_m and ω_m are the motor angle and speed.

The load torque in Equation (8) is still unexpanded and is referred from the cable tension through the screw. Referring the tension of Equation (4) to the motor shaft with lead p_h transfers the cable-side load to the drive side, which gives:

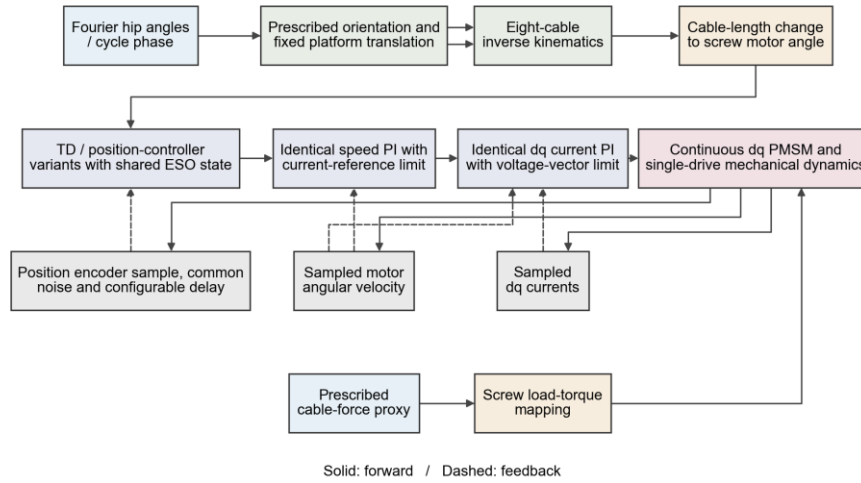
$$T_L = \frac{p_h}{2\pi\eta} F_c \quad (9)$$

125 where F_c is the cable tension and η is the screw efficiency. A screw is a speed reducer, so a modest tension produces a considerable load torque. This is why the robot already approaches its torque limit at the nominal cadence.

Equation (9) gives the referred load torque, and the drive itself imposes current and voltage limits within which any controller must operate. These limits are the basis for judging whether a reference can be executed, and they are written as:

$$i_d^2 + i_q^2 \leq I_{max}^2, \quad u_d^2 + u_q^2 \leq U_{max}^2 \quad (10)$$

130 where I_{max} and U_{max} are the current and voltage limits. At high speed the voltage constraint dominates, whereas at low speed with a heavy load the current constraint takes over.



(a) Reference generation and drive chain

Figure 2. Reference generation and drive chain.

Table 1. Selected model parameters and their provenance.

Symbol	Parameter	Value	Unit	Provenance
p ₁	Actual stator resistance	0.8	ohm	assumed
p ₂	Actual d-axis inductance	0.003	H	assumed
p ₃	Actual q-axis inductance	0.003	H	assumed
p ₄	Actual pole pairs	4	1	assumed
p ₅	Actual permanent-magnet flux	0.08	Wb	assumed
p ₆	Actual rotor inertia	0.003	kg m ²	assumed
p ₇	Actual viscous coefficient	0.0005	N m s/rad	assumed
p ₈	DC link voltage	310	V	assumed
p ₉	Absolute q-axis current-reference limit	12	A	assumed
p ₁₀	Absolute outer speed-command limit	500	rad/s	assumed
p ₁₁	Speed PI proportional gain	0.75	A/(rad/s)	assumed
p ₁₂	Speed PI integral gain	22.5	A/rad	assumed



2.3 From the reference to the drive demand

Equations (1)-(3) map the hip reference to the cable lengths, and Equations (4)-(10) impose the tension, motor and limit constraints. To judge whether a given reference can be executed, the unclipped reference derivatives are substituted into the
 135 mechanical equation of Equation (8) to obtain the demanded torque:

$$\tau_{req} = J\ddot{\theta}^{ref} + B\dot{\theta}^{ref} + T_L \quad (11)$$

where θ^{ref} is the motor-angle reference and T_L follows from Equation (9). If τ_{req} exceeds the limits of Equation (10), the reference cannot be realized and no controller can compensate for it. Three margins are defined from this quantity, for speed, torque and voltage, each as one minus the ratio of the peak demand to the corresponding limit; a negative value means the
 140 demand has crossed the limit.

3 Controller design

3.1 Tracking loop and disturbance observation

Equation (10) gives the limits that the drive must respect, and the tracking law is designed within them. The controller keeps the cascade of an outer position loop and an inner current loop, with the changes concentrated in the position loop, so the
 145 position error and the nominal input are defined as:

$$e = \theta^{ref} - \theta_m, \quad u_0 = k_p e \quad (12)$$

where θ^{ref} is the angle reference, θ_m is the motor angle and k_p is the position gain. The proportional law handles nominal tracking only, while the speed and current loops keep their conventional PI settings, the three forming one chain.

Equation (12) gives the position error and the nominal input, which assures nominal tracking only, while load changes and
 150 parameter deviations have to be estimated separately. Gathering them and the unmodelled dynamics into one lumped disturbance gives the third-order linear extended-state observer:

$$\dot{z}_1 = z_2 + \beta_1 e_o, \quad \dot{z}_2 = z_3 + \beta_2 e_o + b_0 u, \quad \dot{z}_3 = \beta_3 e_o \quad (13)$$

where z_1 , z_2 and z_3 estimate position, speed and the lumped disturbance, $e_o = \theta_m - z_1$ is the observation error, and b_0 is the input gain. Collecting every uncertainty into z_3 avoids item-by-item modelling, at the price that the observer bandwidth must
 155 exceed the frequency band of the disturbance.



The observer of Equation (13) contains three gains that are inconvenient to tune one by one, and bandwidth parameterization reduces them to one. The observer and closed-loop characteristics are then fixed by bandwidth alone, which provides a unified entry for tuning:

$$\beta_1 = 3\omega_o, \quad \beta_2 = 3\omega_o^2, \quad \beta_3 = \omega_o^3, \quad k_p = \omega_c^2 \quad (14)$$

160 where ω_o is the observer bandwidth and ω_c the control bandwidth. If the plant is close to second order in the unsaturated region, the closed loop and the observer reduce to the characteristic forms $s^2 + 2\zeta\omega_c s + \omega_c^2$ and $(s + \omega_o)^3$, both fixed by bandwidth alone. The approximation holds in the unsaturated linear region only; the deviation caused by sampling and limiting is examined by the cases below.

3.2 Switching correction and hysteresis

165 Equation (14) fixes the observer and control bandwidths, while the correction law of the position loop is still to be chosen. Switching has to resolve a contradiction, since the correction should stay smooth for small errors and become forceful for large ones, and the nonlinear correction function serves this purpose:

$$\text{fal}(e, \alpha, \delta) = e\delta^{\alpha-1} \quad (|e| \leq \delta); \quad \text{fal}(e, \alpha, \delta) = |e|^\alpha \text{sgn}(e) \quad (|e| > \delta) \quad (15)$$

170 where α is the nonlinearity and δ the half-width of the linear region. Inside δ the correction is nearly linear, beyond it the correction grows as $|e|^\alpha$.

Equation (15) gives the nonlinear correction, and a mode flag selects between the linear and nonlinear corrections. This links the mode flag to the correction strength and prepares the switching criterion, so the switched correction is:

$$u_c = k_p e \quad (\sigma = 0); \quad \text{or} \quad u_c = k_{pn} \text{fal}(e, \alpha, \delta) \quad (\sigma = 1) \quad (16)$$

175 where k_{pn} is the nonlinear gain and σ the mode flag. If σ were allowed to switch freely at every sample, noise alone would make it chatter, so the switch has to follow a criterion with hysteresis.

Equation (16) selects the correction law by the mode flag, and the criterion must be independent of units, for which the error is first normalized. Two switching variables are defined from the observation error and the feedback error, each updated independently:

$$u = \frac{u_0 - z_3}{b_0} \quad (17)$$

180 where E and D are the corresponding thresholds and e_o and e are the observation and tracking errors. Normalization is what allows one set of thresholds to be used across drives and cadences.



Equation (17) gives the two normalized switching variables, and a mode flag switched point by point would chatter under noise, so switching must follow a rule with hysteresis. The update law of the mode flag is:

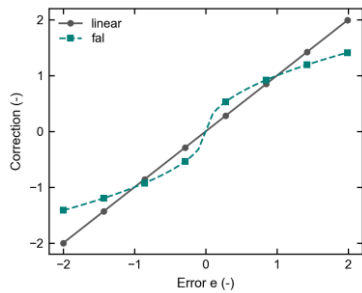
$$q_E = \frac{|e_o|}{E}, \quad q_D = \frac{|e|}{D} \quad (18)$$

185 where h is the hysteresis width, k is the sample index and q stands for either q_E or q_D . Starting from $\sigma=0$, the switching variable must reach unity before the nonlinear mode is entered; starting from $\sigma=1$, it must fall below $1-h$ before the linear mode is resumed. The two variables update independently and govern the observer and feedback corrections separately, and the observer states are not reset at a switch.

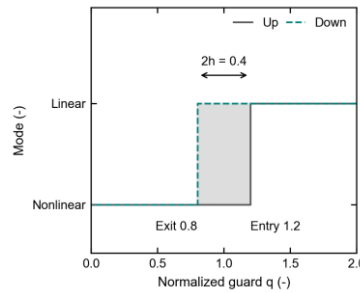
Equation (18) gives the update rule of the mode flag, with the observer states kept unchanged at a switch, and the
 190 disturbance estimate is then compensated. The control input, which is the command actually received by the drive and is limited by Equation (10), is:

$$\sigma(k)=1 \text{ if } [\sigma(k-1)=0 \wedge |q(k)| \geq 1] \vee [\sigma(k-1)=1 \wedge |q(k)| > 1-h]; \quad \sigma(k)=0 \text{ otherwise} \quad (19)$$

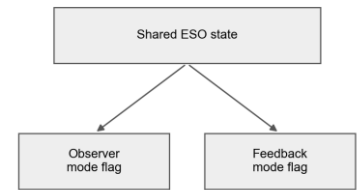
where u_0 is the nominal input and z_3 the disturbance estimate. Equation (19) shows that keeping the observer states does not make the two modes equivalent, because a change in the feedback correction still changes the raw command. The point is
 195 revisited in the ablation of Section 5.5.



(a) Linear and nonlinear correction laws



(b) Switching variable with a hysteresis band of width $2h$



(c) One shared observer state with two mode flags

Figure 3. Construction of the switching law.

3.3 Conditional dwell property of the switching variable

The hysteresis width is quantitatively related to the switching frequency. For a single switching variable q that is continuous and rate-bounded by L_q , the interval between opposite switches satisfies the following relation, which gives the physical meaning of the hysteresis width:

$$\Delta t \geq \frac{2h}{L_q} \quad (20)$$

200

where Δt is the interval between opposite switches and h the hysteresis width. The relation gives the hysteresis width a physical meaning, but it assumes a continuous variable with a bounded rate. Measurement jumps, threshold adjustments and



saturation transients all break that assumption, so the switching count in this paper is counted from the mode history rather than inferred from this relation.

205 3.4 Comparators and tuning effort

Six controller families take part in the comparison. The baseline is a conventional cascade (PID) together with two ADRC families that share one structure and differ only in whether the correction is linear or nonlinear (LADRC, NLADRC). The switching family then has three members: SADRC, which switches without hysteresis, H-SADRC, which is the implementation proposed here, and a continuous variant that replaces the discrete switch by a smooth transition. Two further
 210 variants, keeping only the observer switch or only the feedback switch, appear solely in the matched-gain ablation of Section 5.5. All families were given the same number of candidates and the same tuning effort, listed in Table 2.

Table 2. Controller families and tuning conditions.

Family	Class	Candidates	Runs	Tuning scope
PID	PID-class	54	54	training_four_second_e4dda
LADRC	ADRC-class	54	54	training_four_second_e4dda
NLADRC	ADRC-class	54	54	training_four_second_e4dda
SADRC	ADRC-class	54	54	training_four_second_e4dda
H-SADRC	proposed	0	0	training_four_second_e4dda
Continuous SF	variant	54	54	training_four_second_e4dda
ESO-only	variant	0	0	training_four_second_e4dda
SEF-only	variant	0	0	training_four_second_e4dda

4 Simulation setup and performance metrics

4.1 Tuning and cases

215 The parameters were selected on an independent four-second single-channel training input. The initial step occurs at 0.05 s, from 2 s the command follows a two-term sinusoidal segment, and the load consists of a 1 N m pulse superimposed on a small periodic component. The tuning score combines the step error, the periodic error, the root-mean-square of the periodic q-axis current and the fraction of time during which the current reference is saturated, so that a candidate cannot trade an unacceptable command for tracking accuracy. Two prespecified grids provided 54 candidate evaluations per independently
 220 tuned family, and every case thereafter used fixed gains. The finite grids define the tuning effort tested here and do not represent a global optimum.

The comparison covers 154 cases in three groups. Conventional cases, namely amplitude steps, load disturbance and the cadence sweep, examine transient and steady-state behaviour. Non-stationary eight-channel loads and thirty paired parameter



realizations examine inconsistencies of mechanism and parameters. A 4×4 grid of encoder noise and delay, thirty paired
 225 noise trials and a 64-point scan of thresholds and hysteresis examine measurement conditions and tuning sensitivity. A
 numerical integration-step check closes the set and rules out discretization artefacts. Each case keeps its inputs, configuration
 and diagnostic data, and each figure traces back to its data. For statistics, one seeded trial pooling its eight channels is treated
 as one independent sample; channels and time samples are not counted as independent replicates, which would overstate
 significance by ignoring their correlation.

4.2 Performance metrics

The model and controller above determine the behaviour of the drive in each case, and evaluating it requires a unified metric.
 The five metrics below cover transient speed, overshoot, channel pooling, switching count and paired difference, and the
 settling time is defined as the instant at which the response enters and stays within the 2% error band:

$$t_s = \min\{t: |e(\tau)| \leq 0.02r, \forall \tau \in [t, T]\} \quad (21)$$

235 where r is the step amplitude, T the simulated duration and $e(\tau)$ the tracking error. A response that leaves the band within the
 duration is recorded as not settled rather than being assigned a truncated value.

Equation (21) measures how fast the response is, and the overshoot measures how far the peak exceeds the step amplitude
 relative to that amplitude. Together they describe the transient quality and serve the comparison among controllers:

$$M_p = \frac{\max_{\tau} \theta_m(\tau) - r}{r} \times 100\% \quad (22)$$

240 where $\max \theta_m(\tau)$ is the peak response. For a position servo the overshoot is exactly how far the platform travels past the
 reference, which is closer to what a user feels than the steady-state error.

Equations (21) and (22) concern a single response, while the behaviour of the whole machine requires pooling the eight
 channels. The eight channel errors are pooled as a root-mean-square value:

$$\text{RMSE}_p = \sqrt{\frac{1}{8N} \sum_{i=1}^8 \sum_{k=1}^N e_{i,k}^2} \quad (23)$$

245 where $e_{i,k}$ is the motor position error of channel i at sample k and N is the number of samples. The quantity is a diagnostic of
 the motor layer, not a platform pose error; multi-turn motor error and wrapped phase error are both retained so that the real
 screw travel is not hidden.



Equation (23) gives the pooled channel error, and the switching frequency is counted separately. The number of mode actions is taken directly from the mode history, reflecting how frequently the mode changes and depending on the hysteresis width:

$$N_{sw} = \frac{1}{T} \sum_{k=1}^N \mathbf{1}\{\sigma(k) \neq \sigma(k-1)\} \quad (24)$$

where $\sigma(k)$ is the mode flag at sample k , T is the counting interval and the indicator function picks out the changes. Because discrete mode changes are counted, the metric relies on no assumption beyond the sample step.

Equations (23) and (24) give the pooled error and the switching count, and controllers are compared on the same conditions through paired differences, which remove the influence of trial-to-trial randomness:

$$d_j = \text{RMSE}_p^H(j) - \text{RMSE}_p^B(j), \quad j = 1, \dots, 30 \quad (25)$$

where d_j is the difference of trial j , RMSE_p^H and RMSE_p^B are the pooled errors of the proposed method and the baseline on the same trial, and j runs to 30. The interval of the paired difference is obtained by bootstrapping; an interval that excludes zero indicates a credible difference, which is then compared with the practical thresholds of Table 3, and a difference that is statistically resolvable but below that threshold is treated as negligible.

Table 3. Metrics: definition, unit, counting interval and practical threshold.

Metric	Definition (abridged)	Unit	Interval	Practical threshold
rmse_rad	$\sqrt{\int_{t_0}^{t_1} (\theta - \theta_r)^2 dt}$	rad	case-specific arch	$\text{abs}(\text{difference}) \geq \text{max}$
pooled_rmse_rad	$\sqrt{\frac{1}{8} \sum_{i=1}^8 \text{RMSE}_i}$	rad	periodic second/th	$\text{abs}(\text{difference}) \geq \text{max}$
mean_channel_rmse_rad	$\frac{1}{8} \sum_i \text{RMSE}_i$	rad	ablation [3,9]	$\text{abs}(\text{difference}) \geq \text{max}$
worst_channel_rmse_rad	$\max_i \text{RMSE}_i$	rad	periodic second/th	$\text{abs}(\text{difference}) \geq \text{max}$
mean_rmse_rad	$\frac{1}{N} \sum_{s=1}^N \text{pooledRMSE}_s$	rad	each trial [3,9]	$\text{abs}(\text{difference}) \geq \text{max}$
sd_rmse_rad	$\sqrt{\sum_s (r_s - \bar{r})^2 / (N-1)}$	rad	each trial [3,9]	not an effect-size thr
mean_diff_rad	$\frac{1}{N} \sum_s (r_{H,s} - r_{B,s})$	rad	each paired trial	$\text{abs}(\text{difference}) \geq \text{max}$
settling_s	$\inf\{t-t_s: \theta(u) - r \leq 0.02 r , \forall u \in [t, t+T]\}$	s	step onset 0.05 to	$\text{abs}(\text{difference}) \geq 0.0$

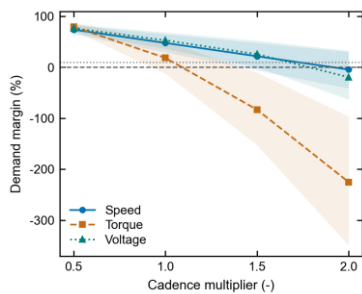
5 Results and discussion

5.1 Capability limits of the individual drives

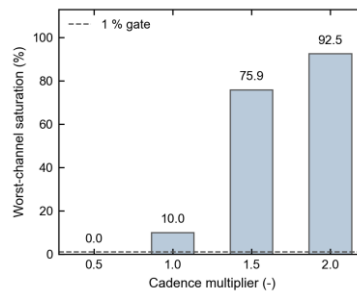
A comparison of controllers presupposes that the reference can be executed at all, and Figure 4 tests that premise. In Figure 4(a) the three margins fall monotonically as the cadence multiplier rises. At half cadence the tightest channel still keeps a torque margin of 72.171%; at the nominal cadence it has dropped to -12.12%, so the demanded torque exceeds the limit by



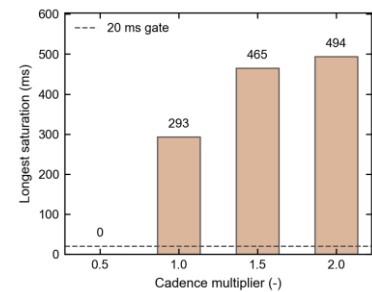
12.12%; the speed margin turns negative beyond 1.5 times the nominal cadence and reaches -41.172% at twice the nominal cadence. The speed margin fails first, which mirrors Equation (6), where the back-electromotive term grows linearly with speed. Figure 4(b) gives the saturated fraction of the worst channel: 10.017% at the nominal cadence, rising to 92.5% at twice the nominal cadence, where the drive spends almost the whole interval in current limit. Figure 4(c) adds a different aspect: saturation is not an instantaneous clip but a limitation lasting several milliseconds, up to 494 ms at twice the nominal cadence, which is beyond the reach of the current loop. The comparison range of this paper follows from these panels: the operating range of the machine is set by its drives, and no controller can supply torque or speed that the reference does not have. The comparisons below are therefore made where the constraints are met or only slightly exceeded, and saturated cases are reported as outcomes of drive capability rather than as a ranking of controllers.



(a) Speed, torque and voltage demand margins



(b) Saturated fraction of the worst channel



(c) Longest continuous saturation

Figure 4. Capability limits of the reconstructed reference.

5.2 Transient and load response

Within that range, the transient at large amplitude separates the two correction laws most clearly. Figure 5 shows the step responses at three amplitudes together with the corresponding metrics, the three trajectory panels being arranged by amplitude so that the trend with amplitude is visible at a glance. In Figure 5(a), at 10 rad, the PID class is fastest and enters the error band in 0.154 s against 0.184 s for H-SADRC, the two differ only slightly, and all curves nearly coincide in the steady part: at small amplitude the benefit of disturbance compensation has not yet appeared. Figure 5(b) at 30 rad looks different. The PID class overshoots by 52.171%, LADRC by 16.008%, whereas H-SADRC overshoots by 0.311% and settles within 0.337 s; NLADRC never enters the band within the observed duration and overshoots by 89.046%. Figure 5(c) at 60 rad widens the gap further: 71.408% for the PID class, 8.038% for LADRC, and still 1.995% for H-SADRC. Seen together, the three panels show that the difference in overshoot does not come from the steady-state gain but from the way the correction behaves at large error: a linear correction keeps its gain everywhere and allows the output to rise unchecked, while a nonlinear correction grows as $|e|^\alpha$, so the equivalent gain falls and the overshoot is suppressed. Settling time and overshoot quantify that difference. In Figure 5(d), plotted on a logarithmic axis, the settling time of the PID class lengthens from 0.154 s to 1.341 s, whereas that of H-SADRC only moves from 0.184 s to 0.549 s; the two slopes differ,



290 which means the proposed loop is less sensitive to the amplitude. Figure 5(e) uses the same logarithmic axis, and the comparison between the 30 rad and 60 rad bars is the clearest: the H-SADRC bars stay close to zero at every amplitude, while the PID class is already past half at 30 rad. For a training robot the platform travelling beyond the reference is transmitted to the trainee directly, so this improvement carries more engineering weight than the steady-state error.

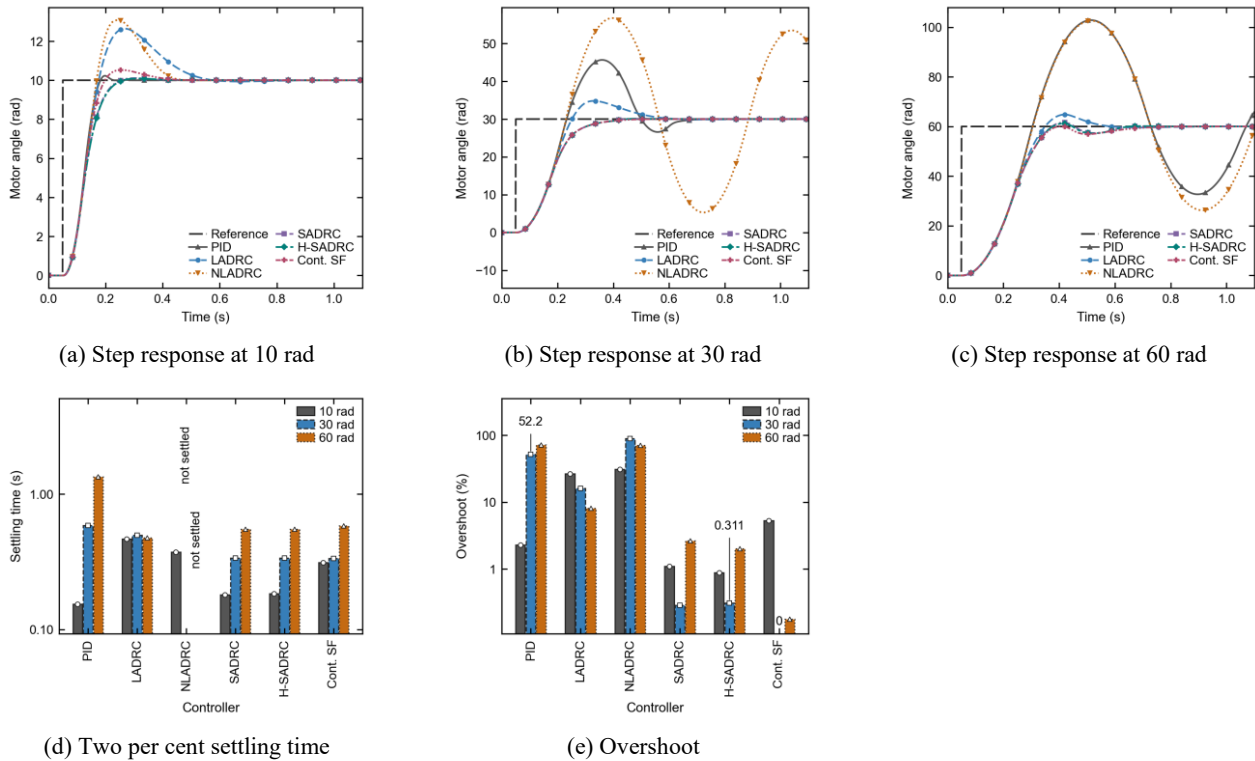


Figure 5. Step responses at fixed gains.

Beyond the transient, a shift of the trainee's weight acts on the drives as a load. Figure 6 is devoted to that situation: a 3 N m
 295 load is applied after the 30 rad step, and the shaded band marks the load interval. Figure 6(a) shows that the differences between curves are concentrated in the first 0.2 s after loading and then decay. Figure 6(b) gives the maximum absolute error in that interval: 0.0447 rad for H-SADRC, 0.0447 rad for SADRC, 0.101 rad for the PID class and 0.15 rad for LADRC. NLADRC reports 19.139 rad, but that number comes mostly from the earlier step transient, and its current trace in Figure 6(c) shows no pronounced jump at the loading instant, so for this family the interval error cannot be read as weak disturbance rejection. Figure 6(c) adds evidence beyond the error itself: the current rise of H-SADRC at the loading instant is about half
 300 that of SADRC and then falls back smoothly, which shows that the compensation acts at the onset of the disturbance instead of relying on steady-state gain. This matters for a cable drive, because a current jump appears at the screw as a tension jump.

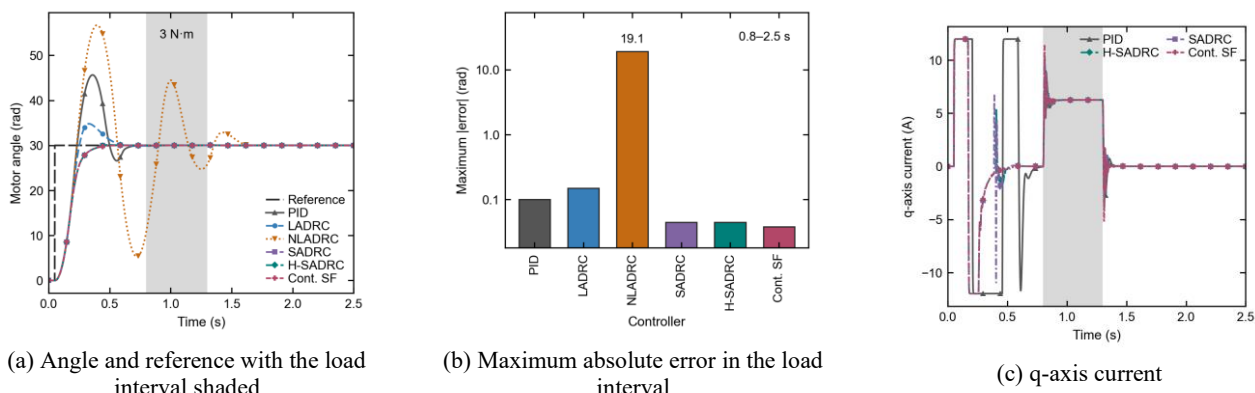
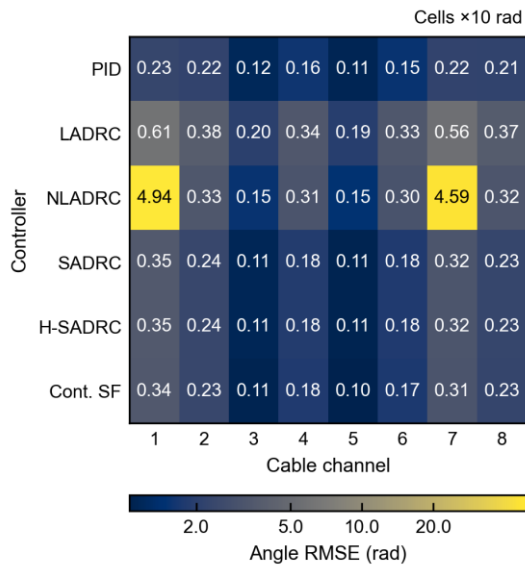


Figure 6. Regulation under load disturbance.

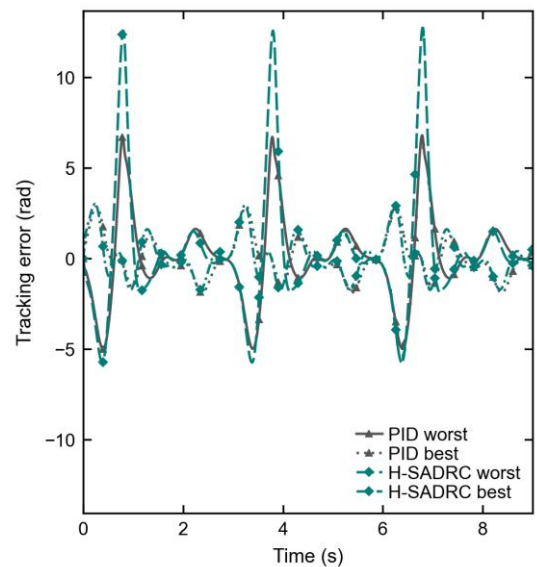
5.3 Multi-channel tracking and cadence

Beyond single-channel error, the spread among channels and the change of cadence jointly determine how well the machine reproduces the motion, and both are discussed here.

Looking at the eight channels separately reveals another layer. Figure 7(a) presents the error of each controller on each channel as a heat map, and within every controller the error on channels 1 and 7 stands clearly above the rest. Those two channels sit near a geometric singularity, where the condition number of the Jacobian is large, so a small error in cable length is magnified when the structure matrix is inverted. Figure 7(b) plots the time traces of the worst and the best channel, and the error of the worst channel is concentrated in the reversal of the push while the constant-speed part matches the other channels. The two panels together give a practical conclusion: the pooled eight-channel error does not represent any single channel, because the spread among channels is set by mechanism geometry rather than by the controller. Improving the whole machine therefore calls for adjusting the cable layout near the singular configuration before raising the gain of a single channel.



(a) Per-channel motor position error



(b) Error traces of the worst and the best channel

Figure 7. Tracking of the eight channels under the nominal periodic reference.

315 Once the cadence rises, the limits identified in Section 5.1 become active. In Figure 8(a) the pooled error of H-SADRC is 0.683 rad at half cadence against 0.903 rad for the PID class. At 1.5 and twice the nominal cadence the error of every family rises sharply, and H-SADRC reaches 20.731 rad and 37.535 rad. The reason is in Figure 8(b): at twice the nominal cadence the saturated fraction of the worst channel reaches 92.5%, the command is clipped for most of the interval, the loop gain collapses and the available voltage, not the control law, becomes the dominant source of error. This confirms the conclusion

320 of Section 5.1 that comparisons outside the feasible range carry no physical meaning; only cases that remain inside the limits, or slightly beyond them, can be used to rank controllers.

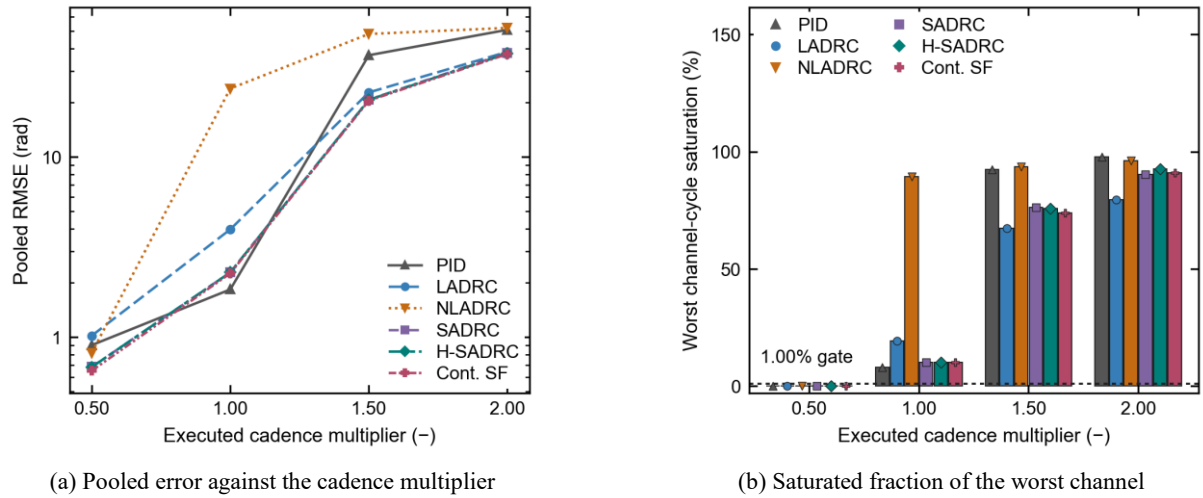


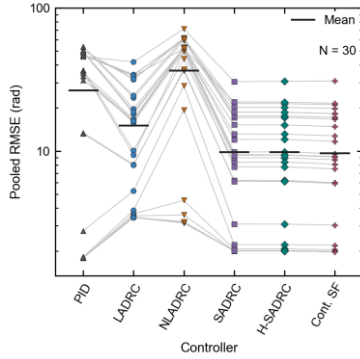
Figure 8. Effect of the cadence multiplier.

5.4 Parameter variation and measurement conditions

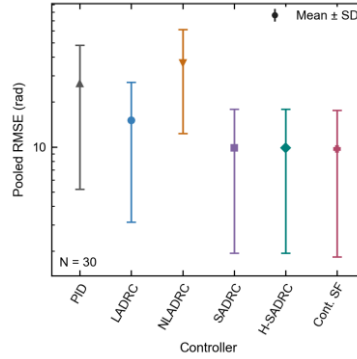
The preceding comparisons rest on nominal parameters and ideal measurements, and this section examines the effect of parameter variation, encoder noise and feedback delay.

325 All comparisons so far rest on nominal parameters, whereas a real machine deviates in resistance, inductance, flux linkage and inertia. Thirty paired realizations, collected in Figure 9, examine this. Figure 9(a) shows the pooled error of each trial as points joined across controllers, so that the four families of one trial sit side by side: the spread between trials far exceeds the differences between controllers, and a few trials exceed 60 rad because their parameters already sit at the capability limit of Section 5.1. Figure 9(b) shows means and standard deviations: 9.878 rad with a standard deviation of 7.945 rad for H-SADRC, 26.592 rad for the PID class and 15.085 rad for LADRC. Figure 9(c) gives the paired differences and bootstrap intervals: against the PID class the difference is -16.714 rad with an interval of [-21.924, -11.387] rad, and H-SADRC is lower in 19 of the thirty trials; against SADRC the difference is -0.00158 rad and the interval contains zero, so with these parameters and thresholds hysteresis does not change tracking accuracy.

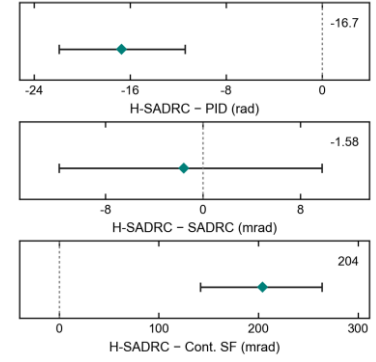
335 The third panel of Figure 9 deserves a separate remark, because it places the proposed implementation alongside the continuous variant of the same family. The paired difference is 0.204 rad, about two per cent of the mean pooled error, a difference that originates from the command jump at a discrete switch. This agrees with Equation (19): keeping the observer states does not make the two modes equivalent, and the command jump at a discrete switch introduces a visible but not dominant error. Given that, the choice between them should not rest on accuracy alone. The continuous variant has no discrete switch, but it also has no adjustable hysteresis; in the proposed implementation the switching count can be set directly through the hysteresis width, which is shown in Section 5.5. For a cable robot the cost of a tension jump usually outweighs a small tracking error, so retaining a controllable switching frequency is of more practical value.



(a) Pooled errors of the thirty paired trials



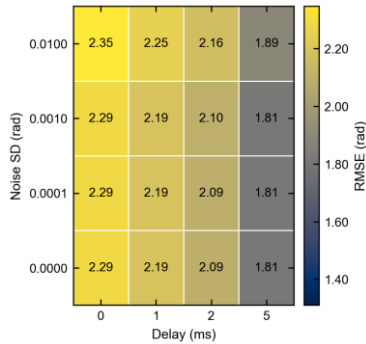
(b) Mean and standard deviation



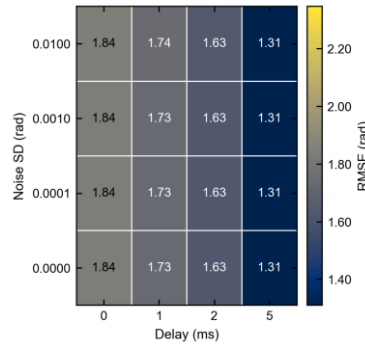
(c) Paired differences with bootstrap intervals

Figure 9. Robustness under paired parameter variation.

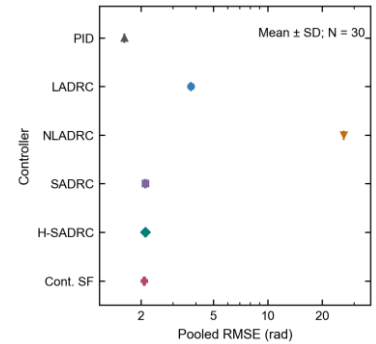
Encoder noise and feedback delay change the quality of the signal that the loop sees. Figure 10(a) and 10(b) give a 4×4 grid for the proposed family and a reference family, with position delay on the horizontal axis and noise standard deviation on the vertical axis. The common trend of both grids is a rise of error with noise, whereas the dependence on delay is not monotonic:
 345 in a few cells a longer delay reduces the periodic error. This is a closed-loop phase effect rather than a benefit of delay, because the delay changes the phase relation between error and correction and in some cases weakens the periodic component. Figure 10(c) gives the quantitative comparison over thirty noise trials: at a noise standard deviation of 0.01 rad and a position delay of 2 ms the pooled error of the PID class is 1.615 rad and that of the switching ADRC family is 2.11 rad. The difference is statistically resolvable but small compared with the transient and load errors of Section 5.2, which shows
 350 that the proposed loop holds its pooled error at 2.110 rad under the same conditions; noise and delay are therefore conditions to be controlled in the measurement chain, not premises of the method.



(a) Noise-delay grid for the proposed family



(b) Noise-delay grid for a reference family



(c) Thirty paired noise trials

Figure 10. Effect of encoder noise and feedback delay.



5.5 Switching count and ablation

Hysteresis turns the switching frequency into a design quantity; this section first verifies this with the hysteresis width and then separates the two switching paths by ablation.

355 The effect of hysteresis is concentrated in the switching count. Figure 11 answers under thresholds of $E = 1$ rad and $D = 1500$ rad/s². In Figure 11(a) the number of mode actions falls quickly with the hysteresis width: 46 actions without hysteresis, 22 at $h = 0.1$, 18 at $h = 0.2$ and 16 at $h = 0.4$, reductions of 52.174%, 60.87% and 65.217% respectively. Figure 11(b) gives the change of error relative to $h = 0$, and the whole curve stays at the 10^{-4} rad level, which is negligible against the step errors of Section 5.2. Taken together the two panels show that hysteresis lowers the switching count while leaving the tracking level unchanged. Figure 11(c) reports two electrical proxies: at $h = 0.2$ the current total variation changes by -0.516% and the copper-loss proxy by -0.173%, both smaller than the change in the switching count, which means that hysteresis first saves a
 360 level unchanged. For device lifetime and electromagnetic interference, the switching frequency is itself the quantity to be controlled.

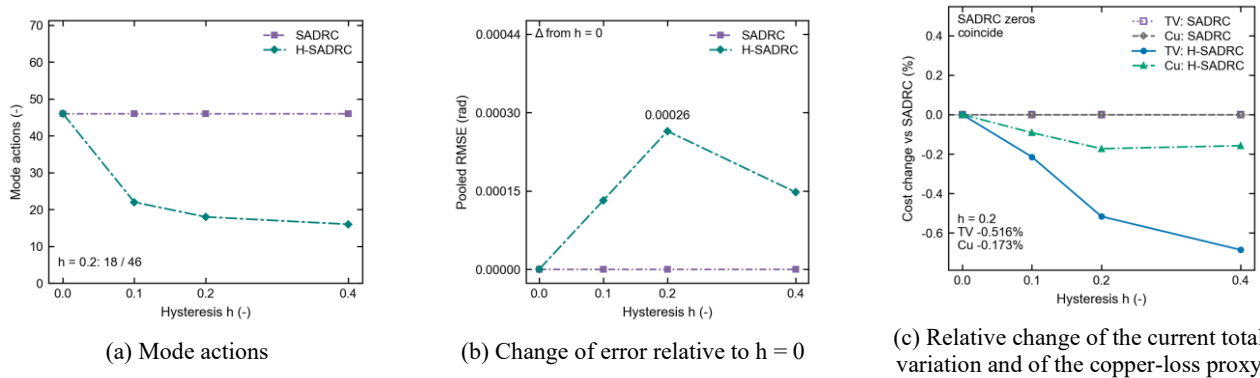


Figure 11. Effect of the hysteresis width on the switching count and on tracking.

Only one of the two switching paths has a substantive effect on the results. Figure 12 keeps the SADRC gains of the source and separates the switches. Figure 12(a) gives the pooled error of three configurations: 2.241 rad with both switches, 27.901 rad with the observer switch only and 2.241 rad with the feedback switch only. The first two are almost identical while the third degrades clearly, so under these thresholds the effective path is the feedback correction and the observer switch contributes little. This is consistent with Section 3.2: the nonlinear correction changes the equivalent gain of the position law, whereas the observer switch changes only the gain of the disturbance estimate, which has limited influence on steady-state
 370 tracking. Figure 12(b) counts mode actions per channel, and the actions concentrate on the few channels that again sit near the geometric singularity; their errors are larger, cross the threshold more often and therefore switch more frequently.

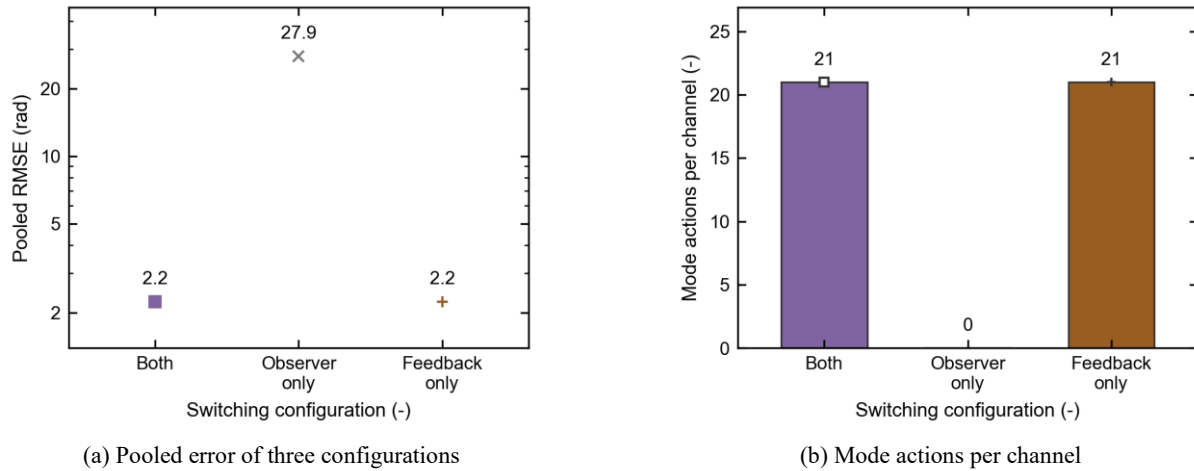


Figure 12. Matched-gain ablation of the switching paths.

6 Conclusions

To meet the requirement that the servo layer of a skating-training robot both reproduce the prescribed reference and limit the switching frequency, this paper proposes a state-preserving hysteretic switching active disturbance rejection control implementation. A model from cable geometry and screw transmission to the permanent-magnet synchronous motor is built, and the speed, torque and voltage margins determine the executable range of the reference; on this basis two normalized switching variables switch the observer and feedback corrections separately, and a hysteresis band constrains the switching frequency. Six controller families are compared at fixed gains with equal tuning effort over 154 cases. The 30 rad step overshoot falls from 52.171% to 0.311%, the maximum error in the load interval falls from 0.101 rad to 0.0447 rad, and the mean pooled error under 30 parameter realizations falls from 26.592 rad to 9.878 rad; at a hysteresis width of 0.2 the number of mode actions falls from 46 to 18. This study provides methodological support and quantitative reference for the parameter design of the servo layer of cable robots.

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