Mech. Sci., 7, 39–47, 2016
www.mech-sci.net/7/39/2016/
doi:10.5194/ms-7-39-2016

Mechanical Sciences
Open Access

A representation of the configurations and evolution of metamorphic mechanisms

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Received: 7 June 2015 – Revised: 5 January 2016 – Accepted: 6 January 2016 – Published: 9 February 2016

Abstract. Metamorphic mechanisms are members of the class of mechanisms that are able to change their configurations sequentially to meet different requirements. The paper introduces a comprehensive symbolic matrix representation for characterizing the topology of one of these mechanisms in a single configuration using general information concerning links and joints. Furthermore, a matrix representation of an original metamorphic mechanism that has the ability to evolve is proposed by uniting the matrices representing all of the mechanism's possible configurations. The representation of metamorphic kinematic joints is developed in accordance with the variation laws of these mechanisms. By introducing the joint variation matrices derived from generalized operations on the related symbolic adjacency matrices, evolutionary relationships between mechanisms in adjacent configurations and the original metmaorphic mechanism are made distinctly. Examples are provided to demonstrate the validation of the method.

1 Introduction

In contrast to a traditional mechanism, a metamorphic mechanism is a mechanism with variable topological structures and it is a good approach for resolving the contradiction between economy, adaptation and efficiency. The concept of metamorphic mechanisms was first introduced based on the idea of reconfiguration in 1996 by Jian S. Dai and Rees Jones, which led to a new era of modern mechanism development (Dai and Rees Jones, 1998).

Research on the metamorphic mechanism has been making significant improvements in fundamentals and applications for nearly twenty years. The essence and characteristics of metamorphic mechanisms as well as three metamorphic approaches including variable components, adjacent relations and kinematic joints were introduced by Dai et al. (2005a) and Liu and Yang (2004). In addition, some of the basic constituent elements of these mechanisms, including links and their connectivity relationships, remain unchanged to give the mechanism's adjacent configuration complex coupling features. These two aspects are key factors affecting the study of methods for configuring metamorphic mechanisms (Zhang et al., 2011). Therefore, to create topological variations in the characteristics of mechanisms in different configurations, the appropriate structural representation for a metamorphic mechanism has been researched in recent years.

Mechanism diagrams, topological graphs and conventional adjacency matrices (Tsai, 2001) are simple and intuitive tools for describing the structure of a mechanism in a single configuration. Dai et al. (2005b) and Dai and Rees Jones (2005) were the first to propose an elementary transformation matrix that represented the variation of a mechanism using the adjacency matrix method. Wang and Dai (2007) introduced joint symbols into the adjacency matrix to express the variations of kinematic joints. In the matrix, all of the links were numbered sequentially and placed in principal diagonal positions; the off-diagonal elements were expressed using joint symbols that represented the connectivity relationship. Lan and Du (2008) used -1 as an element indicating a joint frozen into a new adjacency matrix to represent the topological changes of metamorphic mechanisms. Slaboch and Voglewede (2011) and Korves et al. (2012) proposed mechanism state matrices as a novel way to represent the topological characteristics of planar and spatial reconfigurable mechanisms. These matrices can be used as an analysis tool to automatically determine the degrees of freedom of planar mechanisms that only contain one degree of freedom (DOF) joint. Herve (2006) showed how to create translational parallel manipulators using Lie-group algebra, which can give reference to the related research. Yan and Kang (2009) showed how to perform configuration synthesis of mechanisms with variable topologies using graph theory.

However, the axial orientation of a joint and information on link variations were not epitomized in the aforementioned research. Therefore, Yang (2004) introduced the concept of a geometric constraint for expressing the relative positions and orientations of the joint axes and generalized it into six types: parallelism, coincidence, intersection, perpendicularity, coplanarity and randomness. Li et al. (2010) suggested using a constraint graph from computational geometry rather than the traditional topological graph to characterize a metamorphic linkage to simplify the representation of its configuration changes. The adjacency submatrix of the constraint graph provides a convenient description of changes in the topology of links and joints in the operation of the metamorphic linkage. Li et al. (2009) and Li and Dai (2010a, b) developed a topological representation matrix with information on loops, types of links and joints that included orientation information, which has been used in subsequent research. They also introduced a joint-orientation interchanging metamorphic method based on the matrix.

This paper presents a novel method of characterizing the topology of metamorphic mechanisms in all configurations that involves information about links and joints, including their types and axial orientations. Furthermore, a method of constructing an original matrix that represents the original metamorphic mechanism is proposed. Next, the paper proposes two matrix operations that are useful for representing topological changes and evolving features.

2 Configuration characteristics of metamorphic mechanisms

A metamorphic mechanism is a mechanism with variable topological structures that can be transformed from one structure to another continuously. There are variable parts and coupling parts, giving the metamorphic mechanism a variable topological structure and coupling relationship. In particular, variability is the distinguishing feature that separates metamorphic mechanisms from common mechanisms; this is an important area of research. The incorporation of links, the changing relationships of adjacent links and the changing properties of kinematic pairs have been explored to summarize the variable features of metamorphic mechanisms. In essence, a metamorphic kinematic joint is the essential prerequisite for changing the number of and connective relationships among its active links, leading to a transformation of the configuration of the entire mechanism (Zhang et al., 2011).

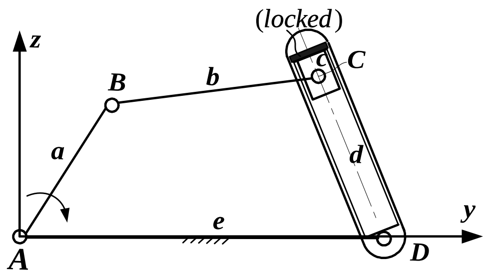
An example of a planar five-bar metamorphic linkage which has five configurations is shown in Fig. 1. Transformations between them are performed by locking different kinematic joints sequentially. When the mechanism is in configuration 1, as shown in Fig. 1a, slider c is locked at the top end of the slot in link d. In this configuration, the mechanism can be treated as a four-bar mechanism. When the mechanism is in configuration 2, as shown in Fig. 1b, links a and b are fixed together by locking the revolute joint B as well as slider c and link d are unlocked. Therefore the mechanism is transformed into a guide bar mechanism. When revolute joint C is locked, links b and c are fixed to transform mechanism into another guide bar mechanism, as shown in Fig. 1c. When the mechanism is in configuration 4, links a and e are fixed together by locking revolute joint A, as shown in Fig. 1d. Link b becomes the driving link of the mechanism. When link d arrives at the location shown in Fig. 1e, joint D is locked to fix links d and e. This transforms mechanism into a crank slider mechanism. Therefore, the mechanism realizes transformations between different configurations by locking its kinematic joints in particular sequences.

From Fig. 1, we conclude that the structure of the mechanism can be transformed from one to another by locking different kinematic joints accordingly. By applying modes such as the geometric limit, force limit, and variation of the driving kinematic joint, the working conditions of these kinematic joints can be switched between active and locked states. In addition, metamorphic kinematic joints are able to change their types and motion orientations to realize configuration transformations (Yan and Kuo, 2006). In a metamorphic mechanism, there is at least one metamorphic kinematic joint, which can change the number and connectivity relationships of active links. There are some basic constituent elements, including links and their connectivity relationships, which remain unchanged to create complex coupling features among the links in adjacent configurations, as shown in Fig. 1.

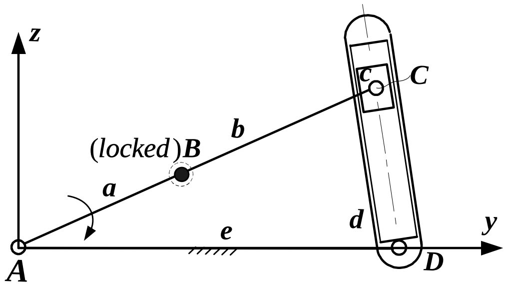
Therefore, to understand the configuration characteristics of metamorphic mechanisms, it is necessary to present a configuration representation that can express not only the characteristics of the mechanism in all of its configurations but also the variations during the transformation process intuitively with the help of specific operations.

3 Representing the configurations of metamorphic mechanisms

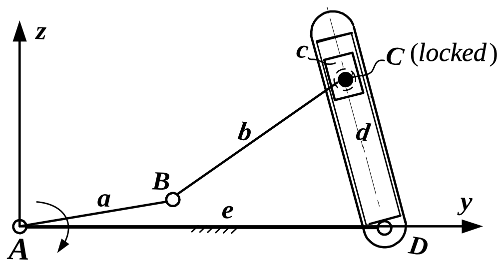
It is known that an adjacency matrix can be used to represent the topological structures of metamorphic mechanisms. This matrix and an *EU*-elementary matrix operation were introduced for expressing a configuration transformation (Dai et al., 2005b; Dai and Rees Jones, 2005). Furthermore, a sym-



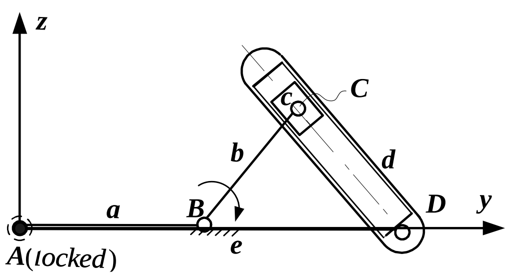
(a) The mechanism in configuration 1



(b) The mechanism in configuration 2



(c) The mechanism in configuration 3



(d) The mechanism in configuration 4

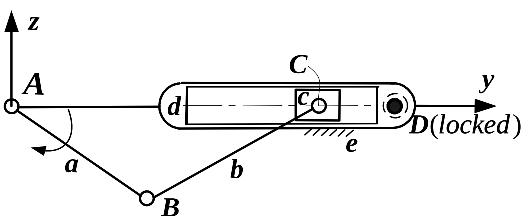


Figure 1. A five-bar planar metamorphic linkage.

bolic adjacency matrix was constructed by introducing information on link variations and joint orientations (Li and Dai, 2010a; Zhang and Ding, 2012). The variations and coupling features of the metamorphic mechanism in adjacent configurations can be determined by applying the generalized difference and intersection operations to the corresponding symbolic matrices.

However, the matrices representing mechanisms in different configurations do not have the same dimension and need to be normalized, increasing the complexity of the representation and operations. Simultaneously, the upper off-diagonal elements in the matrix are the same as the lower off-diagonal elements, which means that the matrix contains information in duplicate. Therefore, to decrease the complexity of expressing the matrix and subsequent operations on it, we improve the symbolic matrix for the mechanism in configuration m and express it as follows:

$$\mathbf{A}^{(m)} = \begin{pmatrix} L_1 & J_{1,2}^{(m)} & \cdots & J_{1,i}^{(m)} & \cdots & J_{1,k-1}^{(m)} & J_{1,k}^{(m)} \\ a_{2,1}^{(m)} & L_2 & \cdots & J_{2,i}^{(m)} & \cdots & J_{2,k-1}^{(m)} & J_{2,k}^{(m)} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\ a_{i,1}^{(m)} & \cdots & \cdots & L_i & \cdots & \cdots & J_{i,k}^{(m)} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ a_{k-1,1}^{(m)} & a_{k-1,2}^{(m)} & \cdots & a_{k-1,i}^{(m)} & \cdots & L_{k-1} & J_{k-1,k}^{(m)} \\ a_{k,1}^{(m)} & a_{k,2}^{(m)} & \cdots & a_{k,i}^{(m)} & \cdots & a_{k,k-1}^{(m)} & L_k \end{pmatrix}, \quad (1)$$

where the principal diagonal element L_i represents the link whose sequence number in the mechanism is i. The numbers of rows and columns are both k, which indicates the number of links in all configurations. Normally, k is greater than or equal to the maximum number of effective links in every configuration. The upper off-diagonal element $J_{i,j}^{(m)}$ denotes the connectivity relationship between links L_i and L_j. It can be represented by a symbol with subscript where the symbol denotes the joint type and the subscript expresses the geometric constraint relationship of the joint axes located at the ends of the link. It is noted that the rule is also applicable for analyzing tertiary links for the essence of the proposed matrix is to record the connectivity relationship between links. A special element -1 is employed here to represent a frozen joint between two links (Lan and Du, 2008) and the element 0 represents the two links that are not connected. Specific expressions for the geometric constraints, including parallelism, intersection, coincidence, perpendicularity and randomicity, are given in Li et al. (2009). The lower off-diagonal element $a_{j,i}^{(m)}$ is the sequence number of the configuration if the state of the corresponding upper off-diagonal element $J_{i,j}^{(m)}$ is changed in configuration m. It should be noted that, for the first configuration matrix $\mathbf{A}^{(1)}$, if there is a joint constraint between links i and j, the value of $a_{j,i}^{(1)}$ is 1. If there is no such constraint, its value is 0. For a matrix $\mathbf{A}^{(m)}$, when only the joint constraint between links i and j is changed in configuration m, the value of $a_{j,i}^{(m)}$ is m. This value is also assigned to the other lower off-diagonal elements to be consistent with the corresponding elements in the previous matrix, $\mathbf{A}^{(m-1)}$. Therefore, dimensional consistency of the matrices for the mechanisms in different configurations is one of the advantages of the proposed symbolic matrix representation. In addition, the symbolic matrix can describe the connectivity relationship of all links synthetically as well as their corresponding variations and provides sufficient information for the subsequent matrix operations.

By applying Eq. (1), we express the five-bar metamorphic linkage, which has the five configurations shown in Fig. 1, as

$$
\begin{aligned}
\mathbf{A}^{(1)} &= \begin{pmatrix} e & R & 0 & 0 & R_{\parallel R} \\ 1 & a & R_{\parallel R} & 0 & 0 \\ 0 & 1 & b & R_{\parallel R} & 0 \\ 0 & 0 & 1 & c & -1 \\ 1 & 0 & 0 & 1 & d \end{pmatrix}, \\
\mathbf{A}^{(2)} &= \begin{pmatrix} e & R & 0 & 0 & R_{\parallel R} \\ 1 & a & -1 & 0 & 0 \\ 0 & 2 & b & R_{\parallel R} & 0 \\ 0 & 0 & 1 & c & P_{\perp R} \\ 1 & 0 & 0 & 2 & d \end{pmatrix}, \\
\mathbf{A}^{(3)} &= \begin{pmatrix} e & R & 0 & 0 & R_{\parallel R} \\ 1 & a & R_{\parallel R} & 0 & 0 \\ 0 & 3 & b & -1 & 0 \\ 0 & 0 & 3 & c & P_{\perp R} \\ 1 & 0 & 0 & 2 & d \end{pmatrix}, \\
\mathbf{A}^{(4)} &= \begin{pmatrix} e & -1 & 0 & 0 & R_{\parallel R} \\ 4 & a & R & 0 & 0 \\ 0 & 3 & b & R_{\parallel R} & 0 \\ 0 & 0 & 4 & c & P_{\perp R} \\ 1 & 0 & 0 & 2 & d \end{pmatrix}, \\
\mathbf{A}^{(5)} &= \begin{pmatrix} e & R & 0 & 0 & -1 \\ 5 & a & R_{\parallel R} & 0 & 0 \\ 0 & 3 & b & R_{\parallel R} & 0 \\ 0 & 0 & 4 & c & P_{\perp R} \\ 5 & 0 & 0 & 2 & d \end{pmatrix},
\end{aligned}
\tag{2}
$$

following the configuration transformation sequence. The numbers of rows and columns in all of these matrices are 5, a result that depends on the number of links a, b, c, d, and e occurring in these five configurations. The upper and lower off-diagonal elements record information on the joint constraints and their variations. For example, comparing $A^{(4)}$ and $A^{(5)}$, the elements $J_{1,2}^{(4)}$, $J_{1,2}^{(5)}$ and $J_{1,5}^{(4)}$, $J_{1,5}^{(5)}$ differ because they show that joint A and joint D have changed from -1 to R and $R_{\parallel R}$ to -1, respectively. Meanwhile, the corresponding elements $a_{2,1}^{(5)}$ and $a_{5,1}^{(5)}$ have changed from 4 to 5 and 1 to 5 to record the sequence number of the configuration in which joints A and D are in these positions in a working cycle.

4 Matrix operations for metamorphic mechanisms

The proposed symbolic matrix describes the topology of the mechanism in a single configuration. However, exploring the variation laws of these mechanisms in different configurations is very important for developing novel metamorphic mechanisms. Therefore, it is feasible to take advantage of matrix operations for constructing the original metamorphic mechanism and determining the features of its topological variations.

4.1 Constructing the original metamorphic mechanism

The original metamorphic mechanism is able to evolve into any configuration of the mechanism and contains all of the topological elements found in all of configurations in a working cycle. A method for constructing original metamorphic mechanisms from biological modeling and genetic evolution was introduced in Wang and Dai (2007) and Zhang et al. (2008). In this paper, based on Eq. (3), an original matrix $\mathbf{A}^{(0)}$ for representing the original metamorphic mechanism is given by

$$
\begin{aligned}
\mathbf{A}^{(0)} &= \mathbf{A}^{(1)} \cup \mathbf{A}^{(2)} \cup \cdots \cup \mathbf{A}^{(m)} \cup \cdots \cup \mathbf{A}^{(n)} \\
&= \begin{pmatrix}
L_1 & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
\vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots \\
\cdots & \cdots & L_i & \cdots & \prod_{m=1}^{n-1} J_{i,j}^{(m)} \cup J_{i,j}^{(m+1)} & \cdots & \cdots \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\
\cdots & \cdots & \left\{a_{j,i}^{(1)}, \cdots, a_{j,i}^{(m)}, \cdots, a_{j,i}^{(n)}\right\} & \cdots & L_j & \cdots & \cdots \\
\vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots & L_k
\end{pmatrix},
\end{aligned}
\tag{3}
$$

where the operator $\cup$ represents the union of its arguments. The result, $\mathbf{A}^{(0)}$, has the same form as Eq. (1). All of the elements located in the same position in the set of related matrices from $\mathbf{A}^{(1)}$ to $\mathbf{A}^{(n)}$ gradually become united, as shown in Eq. (3). Details of the operative principles are as follows:

1. The principal diagonal elements of $\mathbf{A}^{(0)}$ are the same as those of $\mathbf{A}^{(i)}$ ($i = 1, \ldots, n$), indicating the links remain unchanged.

2. The operation that unites the lower off-diagonal elements and records the sequence numbers of the configuration is performed by uniting the elements in these matrices as a set of results in $\mathbf{A}^{(0)}$, which can be expressed as

$$
\mathbf{A}^{(0)}(j,i) = \left\{a_{j,i}^{(1)},\ \ldots,\ a_{j,i}^{(m)},\ \ldots,\ a_{j,i}^{(n)}\right\} \ (i < j \leq k), \tag{4}
$$

where the number 0 is ignored. If the values of the adjacent elements are same in this set, only one of them should be kept. The information given by this set is very helpful for constructing the matrices for a single configuration of the mechanism.

3. The physical meaning of uniting the upper off-diagonal elements of these matrices is to achieve the most variability in the kinematic joints. The operation starts from the upper off-diagonal elements in the first matrix, $\mathbf{A}^{(1)}$; then, the joint type and orientation are expanded based on the elements of the next adjacency configuration matrix in the sequence. We express the operation as

$$\mathbf{A}^{(0)}(i,j)=\prod_{m=1}^{k-1}\mathbf{A}^{(m)}(i,j)\cup\mathbf{A}^{(m+1)}(i,j)$$
$$=\prod_{m=1}^{k-1}J_{i,j}^{(m)}\cup J_{i,j}^{(m+1)}\ (i<j\leq k). \tag{5}$$

Basically, the uniting operator is equivalent to an extension of the type and axial orientation of a kinematic joint. If the adjacent elements are same, it represents the corresponding connectivity relationship between the related links keeps unchanged. So these same numbers in the operation result need to be omitted just keeping one.

For example, according to Eqs. (2)–(5), elements $A^{(0)}(4,3)$ and $A^{(0)}(3,4)$ of matrix $A^{(0)}$ can be calculated as follows:

$$A^{(0)}(4,3)=\{1,1,3,4,4\}=\{1,3,4\} \tag{6}$$

$$A^{(0)}(3,4)=\prod_{m=1}^{4}J_{3,4}^{(m)}\cup J_{3,4}^{(m+1)}$$
$$=R_{\parallel R}\cup R_{\parallel R}\cup-1\cup R_{\parallel R}\cup R_{\parallel R}$$
$$=R_{\parallel R}\cup-1\cup R_{\parallel R}. \tag{7}$$

Therefore, the joint between links b and c changes twice during the configuration transformations from 1 to 3 and from 3 to 4 while its axial orientation remains unchanged during the working cycle.

The construction procedure of the metamorphic kinematic joints can be illustrated in Fig. 2. Firstly, the joints should be listed according to the sequence indicated in the corresponding operation result. Further, geometric limit is used to realize the transformations between these adjacent joints in sequence. Geometric limit is a most common way of making the type of kinematic joints to be changed by releasing or adding appropriate constraints at suitable geometric locations. Such as in Fig. 2a, the kinematic joint between links a and b is a revolute joint whose axis is parallel to the adjacent revolute joint, R_1. In the next configuration, the revolute joint is locked. Therefore, two limiting stoppers are laid on the two links a and b, respectively. When the two stoppers are contacted, the two links are fixed together and the number of DOF of the revolute joint is changed to zero in Fig. 2a. Figure 2b shows that the joint is performing translating motions with arrows denoting the direction of pin's motion and indicating the number of DOFs the joint possesses. When the pin reaches the position shown in the second figure, it stops translating but remains rotating as shown. This is identified as a typical metamorphic kinematic joint that varies from a prismatic joint to a rotating pair. Similarly, Fig. 2c demonstrates a series of varying orientations of a revolute pair undergoing the orientations about different axes, successively.

According to the construction process described above, the matrix of the original metamorphic mechanism for the five-bar metamorphic linkage shown in Fig. 1 is

$$\mathbf{A}_0=\begin{pmatrix} e & R\cup-1\cup R & 0 & 0 & R_{\parallel R}\cup-1 \\ \{1,5\} & a & R_{\parallel R}\cup-1\cup R_{\parallel R} & 0 & 0 \\ 0 & \{1,2,3\} & b & R_{\parallel R}\cup-1\cup R_{\parallel R} & 0 \\ 0 & 0 & \{1,3,4\} & c & -1\cup P_{\perp R} \\ \{1,5\} & 0 & 0 & \{1,2\} & d \end{pmatrix}. \tag{8}$$

Therefore, the original metamorphic mechanism can be generated by applying the uniting operator to all of the mechanism's configurations and using link and joint information. In particular, the matrix which includes the information of all links and their connectivity relationships can make us identify all possible combinations between links for creating different mechanisms. So the mechanism is helpful to develop novel metamorphic mechanisms using the representation method.

4.2 The joint variation matrix

The essential method for realizing configuration transformation of metamorphic mechanisms is to change the characteristics of kinematic joints, which lead to variations in the topology of the entire mechanism. Therefore, to determine the joint variation rule for two adjacent configurations of a mechanism, a generalized difference operation for two adjacency matrices is proposed.

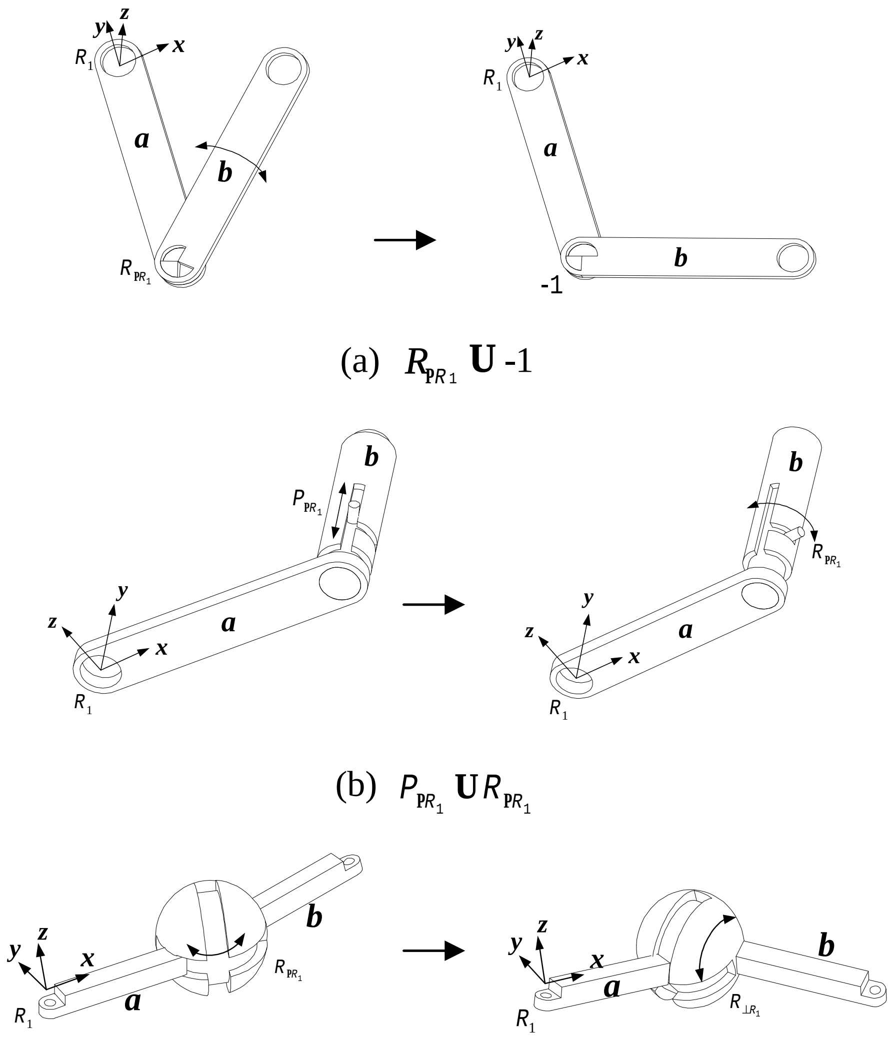
Let $\mathbf{A}_{\mathrm{var}}^{(m+1,m)}$ be the joint variation matrix, which can be described as the result of applying the generalized difference operator to the topological representation matrices $\mathbf{A}^{(m+1)}$ and $\mathbf{A}^{(m)}$, that is

$$\mathbf{A}_{\mathrm{var}}^{(m+1,m)}=\mathbf{A}^{(m+1)}-\mathbf{A}^{(m)}, \tag{9}$$

where – represents the generalized difference operator (Lan and Du, 2008; Li et al., 2010). The resulting matrix contains information about the joint variation when the mechanism is transformed from configuration m to configuration $m+1$. If the mechanism is transformed from configuration $m+1$ to configuration m, the joint variation matrix $\mathbf{A}_{\mathrm{var}}^{(m,m+1)}$ can be expressed as

$$\mathbf{A}_{\mathrm{var}}^{(m,m+1)}=\mathbf{A}^{(m)}-\mathbf{A}^{(m+1)}. \tag{10}$$

The purpose of this operation is to record variations in the upper and lower off-diagonal elements. And the joint variation matrix achieved is the key procedure for constructing the matrix represents the original metamorphic mechanism which will be discussed in Sect. 4.3. If the two elements located at the same position in matrices $\mathbf{A}^{(m)}$ and $\mathbf{A}^{(m+1)}$ are



(a) R_{PR_1} **U** -1

(b) P_{PR_1} **U** R_{PR_1}

(c) R_{PR_1} **U** $R_{\perp R_1}$

Figure 2. The result of uniting different kinematic joints.

equal, the corresponding element in matrix $\mathbf{A}_{\mathrm{var}}^{(m+1,m)}$ is assigned the number 0. Conversely, the elements in the minuend matrix are reserved directly, and the principal diagonal elements remain unchanged for recognition purposes. The same rule is used for the lower off-diagonal elements. If an upper off-diagonal element is unchanged, the corresponding lower off-diagonal element needs to be assigned the value 0 regardless of its actual value. The joint variation matrix can be constructed directly from the physical meaning of the joint variation rule. In addition, analysing the existing joints with the characteristic of metamorphosis is one of the most important approaches for achieving the principle of constructing the corresponding joint variation matrix.

Therefore, joint variation matrices for configurations 1 to 5 are given as follows:

$$\mathbf{A}_{\mathrm{var}}^{(2,1)} = \begin{pmatrix} e & 0 & 0 & 0 & 0 \\ 0 & a & -1 & 0 & 0 \\ 0 & 2 & b & 0 & 0 \\ 0 & 0 & 0 & c & P_{\perp R} \\ 0 & 0 & 0 & 2 & d \end{pmatrix},$$

$$\mathbf{A}_{\mathrm{var}}^{(3,2)} = \begin{pmatrix} e & 0 & 0 & 0 & 0 \\ 0 & a & R_{\parallel R} & 0 & 0 \\ 0 & 3 & b & -1 & 0 \\ 0 & 0 & 3 & c & 0 \\ 0 & 0 & 0 & 0 & d \end{pmatrix},$$

$$\mathbf{A}_{\mathrm{var}}^{(4,3)} = \begin{pmatrix} e & -1 & 0 & 0 & 0 \\ 4 & a & 0 & 0 & 0 \\ 0 & 0 & b & R_{\parallel R} & 0 \\ 0 & 0 & 4 & c & 0 \\ 0 & 0 & 0 & 0 & d \end{pmatrix},$$

$$\mathbf{A}_{\mathrm{var}}^{(5,4)} = \begin{pmatrix} e & R & 0 & 0 & -1 \\ 5 & a & 0 & 0 & 0 \\ 0 & 0 & b & 0 & 0 \\ 0 & 0 & 0 & c & 0 \\ 5 & 0 & 0 & 0 & d \end{pmatrix}. \tag{11}$$

4.3 The relationship between the original metamorphic mechanism and the mechanism in any configuration

Because an original metamorphic mechanism provides a foundation for a mechanism to transform itself into any configuration and expresses the joint variation characteristics from the symbolic adjacency matrices and the corresponding operations, the relationships between these matrices is as shown in Fig. 3.

1. The relationship between adjacent configurations: the two adjacent matrices shown in Fig. 3 can be transformed into each other using a joint variation matrix. From Eq. (9), the matrix $\mathbf{A}^{(m+1)}$ can be expressed as

   $$\mathbf{A}^{(m+1)} = \mathbf{A}^{(m)} + \mathbf{A}_{\mathrm{var}}^{(m+1,m)}, \tag{12}$$

   where $+$ represents the generalized addition operator, which changes the elements in matrix $\mathbf{A}^{(m)}$ according to the corresponding elements in the joint variation matrix of $\mathbf{A}_{\mathrm{var}}^{(m+1,m)}$. Comparing the corresponding elements in the two matrices, the lower off-diagonal elements in $\mathbf{A}_{\mathrm{var}}^{(m+1,m)}$ containing the value m are selected, with the corresponding symmetrical upper triangular elements, to replace the corresponding elements in matrix $\mathbf{A}^{(m)}$ while leaving the other elements unchanged. Similarly, matrix $\mathbf{A}^{(m)}$ can be expressed as

   $$\mathbf{A}^{(m)} = \mathbf{A}^{(m+1)} + \mathbf{A}_{\mathrm{var}}^{(m,m+1)}. \tag{13}$$

   For example, the relationship between matrices $A^{(1)}$ and $A^{(2)}$ is

   $$\mathbf{A}^{(2)} = \mathbf{A}^{(1)} + \mathbf{A}_{\mathrm{var}}^{(2,1)} \tag{14}$$

   $$\mathbf{A}^{(1)} = \mathbf{A}^{(2)} + \mathbf{A}_{\mathrm{var}}^{(1,2)}. \tag{15}$$

2. The relationships of the original metamorphic mechanism and the mechanism in a single configuration: the original metamorphic mechanism is able to evolve into any configuration. Therefore, the information on the mechanism in configuration m can be extracted from the matrix $\mathbf{A}^{(0)}$ to construct the corresponding matrix $\mathbf{A}^{(m)}$. The process of evolution from $\mathbf{A}^{(0)}$ to $\mathbf{A}^{(m)}$ follows from Eq. (3).

First, the principal diagonal elements denoting the links in $\mathbf{A}^{(0)}$ are placed in their corresponding positions in $\mathbf{A}^{(m)}$ directly. Then, the lower off-diagonal elements containing the value 1 and their corresponding upper off-diagonal elements, which represent constraints on the joints of links in matrix $\mathbf{A}^{(0)}$, are similarly mapped to positions in $\mathbf{A}^{(m)}$ as long as the value of the corresponding element is not m. The next important step is to select a number m from the elements comprising sets of numbers and then, to identify its sequence number in the set $\{a_{j,i}^{(1)}, \ldots, a_{j,i}^{(m)}, \ldots, a_{j,i}^{(n)}\}$. The sequence number can be used to determine the corresponding joint constraint conveniently using the element $\prod_{m=1}^{n-1} J_{i,j}^{(m)} \cup J_{i,j}^{(m+1)}$. These elements are then placed into $\mathbf{A}^{(m)}$, the other elements of which are assigned a value of 0.

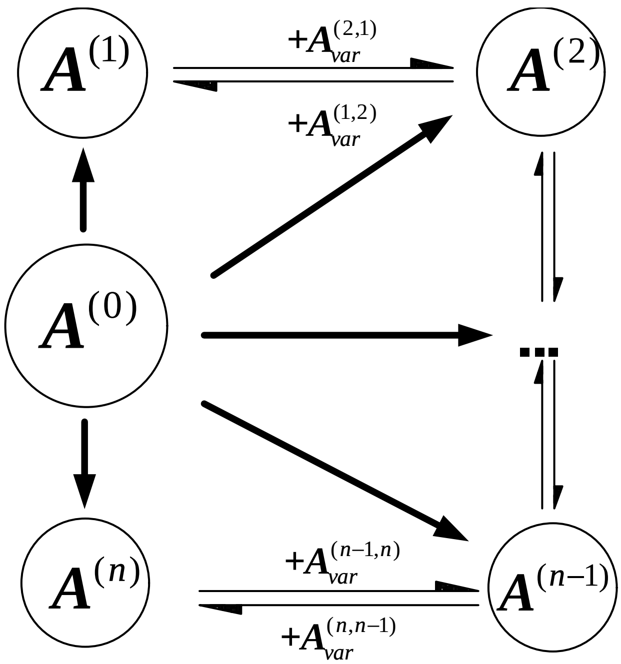


Figure 3. The relationship between the original metamorphic mechanism and the mechanism in any configuration.

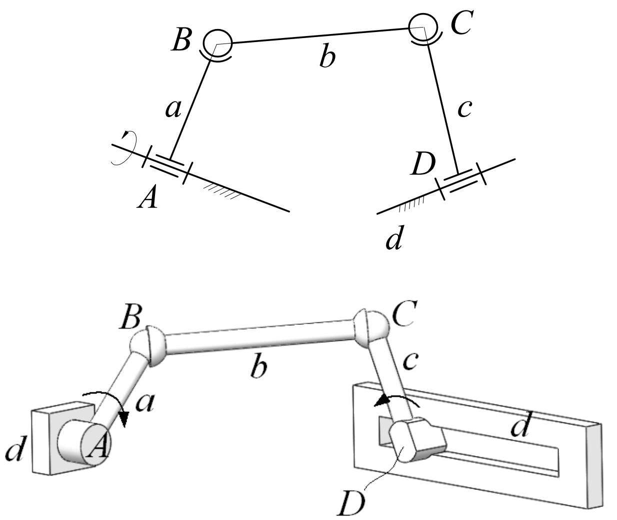
For example, the elements marked by black triangles $\blacktriangledown$ in Eq. (16) are extracted to construct the matrix $\mathbf{A}^{(2)}$, which represents the topology of the mechanism in configuration 2 according to the above procedure.

$$\mathbf{A}_0 = \begin{pmatrix} \underset{\blacktriangledown}{e} & \underset{\blacktriangledown}{R} \cup -1 \cup R & 0 & 0 & \underset{\blacktriangledown}{R_{\parallel R}} \cup -1 \\ \{\underset{\blacktriangledown}{1}, 5\} & \underset{\blacktriangledown}{a} & R_{\parallel R} \cup \underset{\blacktriangledown}{-1} \cup R_{\parallel R} & 0 & 0 \\ 0 & \{1, \underset{\blacktriangledown}{2}, 3\} & \underset{\blacktriangledown}{b} & \underset{\blacktriangledown}{R_{\parallel R}} \cup -1 \cup R_{\parallel R} & 0 \\ 0 & 0 & \{\underset{\blacktriangledown}{1}, 3, 4\} & \underset{\blacktriangledown}{c} & -1 \cup \underset{\blacktriangledown}{P_{\perp R}} \\ \{\underset{\blacktriangledown}{1}, 5\} & 0 & 0 & \{1, \underset{\blacktriangledown}{2}\} & \underset{\blacktriangledown}{d} \end{pmatrix} \tag{16}$$

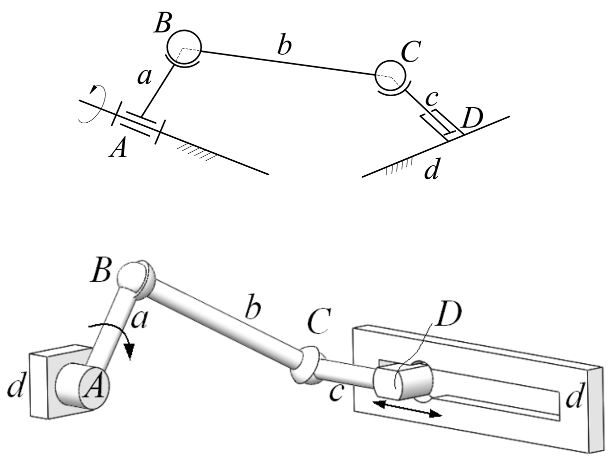
The diagram in Fig. 3 shows that the evolutionary relationships between the original metamorphic mechanism and all of its configurations can be determined by applying matrix operations to the appropriate matrices.

5 Case study

A spatial four-bar metamorphic mechanism that has two configurations is shown in Fig. 4. When the mechanism is in configuration 1, as shown in Fig. 4a, it can be treated as an



(a) The mechanism in configuration 1



(b) The mechanism in configuration 2

Figure 4. A four-bar spatial metamorphic mechanism.

RSSR mechanism. The axis of joint D between links c and d is perpendicular to the axis of joint A between links a and d. When revolute joint D is transformed into a prismatic joint, the mechanism becomes an RSSP mechanism, as shown in Fig. 4b.

The topological structures of the metamorphic mechanism can be expressed in matrix form as follows:

$$\mathbf{A}^{(1)} = \begin{pmatrix} d & R & 0 & R_{\perp R} \\ 1 & a & S & 0 \\ 0 & 1 & b & S \\ 1 & 0 & 1 & c \end{pmatrix} \tag{17}$$

$$\mathbf{A}^{(2)} = \begin{pmatrix} d & R & 0 & P_{\parallel R} \\ 1 & a & S & 0 \\ 0 & 1 & b & S \\ 2 & 0 & 1 & c \end{pmatrix}. \tag{18}$$

The origin matrix of the original metamorphic mechanism and the joint variation matrix can be expressed as

$$\mathbf{A}_{\text{var}}^{(2,1)} = \mathbf{A}^{(2)} - \mathbf{A}^{(1)} = \begin{pmatrix} d & 0 & 0 & R_{\perp R} \\ 0 & a & 0 & 0 \\ 0 & 0 & b & 0 \\ 2 & 0 & 0 & c \end{pmatrix} \tag{19}$$

$$\mathbf{A}^{(0)} = \mathbf{A}^{(1)} \cup \mathbf{A}^{(2)} = \begin{pmatrix} d & R & 0 & R_{\perp R} \cup P_{\parallel R} \\ 1 & a & S & 0 \\ 0 & 1 & b & S \\ \{1,2\} & 0 & 1 & c \end{pmatrix}. \tag{20}$$

The element $R_{\perp R} \cup P_{\parallel R}$ in matrix $\mathbf{A}^{(0)}$ represents the way in which both the axial orientation and the type of joint D have changed. There, the joint can be considered a metamorphic kinematic joint and be developed according to the variation sequence for the kinematic behaviours of the entire mechanism.

6 Conclusions

The paper proposed a comprehensive symbolic matrix for characterizing the topology of a metamorphic mechanism that involved information on the variations of links and the axial orientations of the kinematic joints. In addition, operations on the matrices of the adjacent configuration mechanisms are defined to construct an origin matrix and joint variation matrices. In particular, the construction and evolution of the matrix representation for an original metamorphic mechanism show how it can be transformed into any configuration matrix. The relationship between the original metamorphic mechanism and all of its possible configurations and methods of moving between them were presented. Examples illustrate the effectiveness of this approach in characterizing metamorphic mechanisms. The configuration representation of metamorphic mechanisms provides a foundation for the analysis and synthesis of novel metamorphic mechanisms.

Acknowledgements. The authors gratefully acknowledge the support of the National Natural Science Foundation of China (Project No. 51575018, No. 51275015 and No. 51175494) and the Foundation of State Key Laboratory of Robotics (No. 2014).

Edited by: J. Schmiedeler
Reviewed by: two anonymous referees

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