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Vector model of the timing diagram of automatic machine

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Abstract. In this paper a vector model of timing diagram of automatic machine is developed, which allows us to solve a variety dynamic tasks by changing the parameters of timing diagram of its mechanisms. The connection between the parameters of the timing diagram of automatic machine and equations of motion mechanisms through functions of position and transfer functions of mechanisms is established. The vector model of timing diagram can be used to optimize the timing diagrams of looms and polygraphic machines.

1 Introduction

Modeling of the timing diagram is one of the main parts for design of automatic machines. A detailed analysis of the works on the theory of the timing diagram performed prior to 1965 is given in Petrokas (1970). Timing diagram is a sequence of machine operations performed by mechanisms depending on the angular displacement of the main shaft (Browne, 1965; Youssef and El-Hofy, 2008; Homer, 1998; Sandler, 1999; Natale C, 2003; Singh and Bhattacharya, 2006; Levner and Kats, 2007; Niir Board, 2009; Norton, 2009; Topalbekiroglu and Celik, 2009). Timing diagram allows determining of the position of each of the executive body at any position of the main shaft (see Fig. 1).

Timing diagram automatic machine is modeled by a directed graph (see Fig. 2) (Novgorodtsev, 1982). The disadvantages of this model are the lack of consideration for connections executive bodies displacements mechanisms and accounting precision of manufacturing.

Analysis of the methods of synthesis and analysis of timing diagram automatic machine showed the need for further development of optimization methods of timing diagram, taking into account the accuracy of manufacture and the dynamics of automatic machine.

2 Vector model of the timing diagram of automatic machine

The timing diagram of automatic machines can be represented as the vector polygons (Jomartov, 2010, 2011) (see Fig. 3). Let us replace the segments linear timing diagram by the vectors $\boldsymbol{\ell}_{ij}$. The vectors $\boldsymbol{\ell}_{ij}$ is directed sequentially from one position to another position of mechanism, where i is the number of mechanisms, j is the number of position of i-mechanism, m_j is the number of positions of i-mechanism, n is the number of mechanisms.

The projection of vectors $\boldsymbol{\ell}_{ij}$ on the x axis is α_{ij} – phase angles of actuation of mechanisms. The projection $\boldsymbol{\ell}_{ij}$ on the y axis is the displacement δ_{ij} of j-position of i-mechanism.

$$\delta_{ij} = \frac{S_{ij}}{S_{\max}}, \quad S_{\max} = \max S_{ij}, \quad i = 1, \ldots, n; \; j = 1, \ldots, m_i,$$

where S_{ij} is the displacement of j-position of i-mechanism.

To explain the parameters α_{ij} and S_{ij} in Fig. 4 shows a diagram of the displacement of mechanism, in the figure denote: α_o – phase angles of actuation of mechanism in the position of open, α_d is the phase angles of actuation of mechanism in the position of dwell, α_c is the phase angles of actuation of mechanism in the position of close, S_o is the displacement of mechanism in the position of open, S_c is the displacement of mechanism in the position of close.

Let us introduce the vector $\boldsymbol{P}$ connecting the point of beginning and end of the cycle. The projection of the vector $\boldsymbol{P}$ on the x axis is 2π on the y axis is zero. The interaction of mechanisms with each other will reflect in the form of the vectors of connection $\boldsymbol{c}_{ik}$, where $k = 1, \ldots, r_i$, r_i is the number of vectors of connection of i-mechanism. The projection of the vectors of connection to the x axis is the delay of actuation mechanism, and the projection on the y axis is the difference between the displacements of mechanisms.

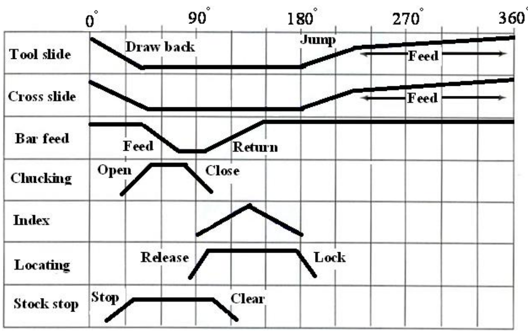

Figure 1. Linear timing diagram of working and auxiliary cams of a four-spindle bar automatic.

Let us impose the timing diagram of mechanisms at each other using zero vectors (see Fig. 3) connecting the boundary points of timing diagram mechanisms in the y axis.

Let us compose the system of vector equations describing the works of mechanisms automatic machine (see Fig. 3).

$$\left.\begin{aligned} &\sum_{j=1}^{m_i} \boldsymbol{\ell}_{ij} = \boldsymbol{P},\ i = 1,\ \ldots,\ n, \\ &\boldsymbol{c}_{ik} = \sum_{i=1}^{n}\sum_{j=1}^{m_i} b_{ij}\cdot \boldsymbol{\ell}_{ij} \end{aligned}\right\}, \tag{1}$$

where $b_{ij} \in \{0,\ \pm 1\}$.

Let us projected the vector Eq. (1) on the axis x and y.

$$\left.\begin{aligned} &\sum_{j=1}^{m_i} \alpha_{ij} = 2\pi,\ \sum_{j=1}^{m_i} \delta_{ij} = 0, \\ &c_{ik}^{x} = \sum_{i=1}^{n}\sum_{j=1}^{m_i} b_{ij}\alpha_{ij},\ c_{ik}^{y} = \sum_{i=1}^{n}\sum_{j=1}^{m_i} b_{ij}\delta_{ij} \end{aligned}\right\} \tag{2}$$

On the phase angles of actuation of mechanisms α_{ij}, and displacements of mechanisms δ_{ij} impose constraints

$$\alpha_{ij} \geq \alpha_{ij}^{\mathrm{m}},\ \delta_{ij}^{\ell} \geq \delta_{ij} \geq \delta_{ij}^{\mathcal{H}}, \tag{3}$$

where α_{ij}^{m} is the minimum allowable phase angles of actuation of mechanisms, δ_{ij}^{ℓ}, $\delta_{ij}^{\mathcal{H}}$ is the upper and lower limits assigned by the designer.

On the projection vectors of connection impose constraints

$$\boldsymbol{c}_{ik}^{x\ell} \geq \boldsymbol{c}_{ik}^{x} \geq \boldsymbol{c}_{ik}^{x\ell},\ \boldsymbol{c}_{ik}^{y\ell} \geq \boldsymbol{c}_{ik}^{y} \geq \boldsymbol{c}_{ik}^{y\mathcal{H}} \tag{4}$$

where $\boldsymbol{c}_{ik}^{x\mathcal{H}} = \boldsymbol{e}_{ik}^{x} + \Delta\boldsymbol{c}_{ik}^{x}$, $\boldsymbol{c}_{ik}^{y\mathcal{H}} = \boldsymbol{e}_{ik}^{y} + \Delta\boldsymbol{c}_{ik}^{y}$ where $\boldsymbol{e}_{ik}^{x}$, $\boldsymbol{e}_{ik}^{y}$ are the minimum permissible projection vectors of connection, $\Delta\boldsymbol{c}_{ik}^{x}$, $\Delta\boldsymbol{c}_{ik}^{y}$ are the errors of the projections of vectors of connection, $\boldsymbol{c}_{ik}^{x\ell}$, $\boldsymbol{c}_{ik}^{y\ell}$ are the upper limits imposed by the designer.

Equation (2) and constraints (Eqs. 3 and 4) describe the collaboration works of mechanisms (timing diagram) of automatic machine.

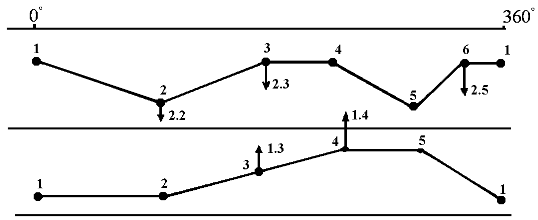

Figure 2. Presentation of timing diagram automatic machine as a directed graph.

3 A mathematical model of automatic machine based on the timing diagram of mechanisms

Lets define the connection between the differential equations of motion the automatic machine and the equations describing its timing diagram. In Fig. 5 shows the dynamic model of the machine, where c_i is the elasticity coefficients, β_i is the coefficients of resistance, J_{i}, I_{i} are the moments of inertia, $M_{\partial B}$ is the motor torque, M_i is moment of resistance, Π_i, $i = 1, \ldots, n$ is the function of position of mechanisms.

To compile the equations of motion mechanisms automatic machine (see Fig. 5), let us use Lagrange equations II (Wolfson, 1976):

$$\left.\begin{aligned} &\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial T}{\partial \dot{\varphi}_j}\right) - \frac{\partial T}{\partial \varphi_j} + \frac{\partial V}{\partial \varphi_j} = Q_j + \sum_{i=1}^{m} \lambda_i h_{ij} \\ &\sum_{j=1}^{m+n} h_{ij}\dot{\varphi}_j + h_i = 0, \end{aligned}\right\} \tag{5}$$

where $\varphi_1,\ \varphi_2,\ \ldots,\ \varphi_n$ are the generalized coordinates, λ_i is Lagrange multipliers, h_{ij}, h_i are some functions, T is the kinetic energy of a holonomic system, V is the potential energy of the system, Q_j is the generalized force.

To establish the connection between the equations of timing diagram (2–4) and the dynamic Eq. (5), let us write the functions of position, the transfer functions of mechanisms of automatic machine in the following form:

$$\left.\begin{aligned} &\Pi_i = \Pi_{i1}\cdot[1 - L(\phi_i - \alpha_{i1})] + \sum_{j=2}^{m} \Pi_{ij}\left[1 - L\left(\phi_i - \sum_{r=1}^{j}\alpha_{ir}\right)\right]\cdot L\left(\phi_i - \sum_{r=1}^{j-1}\alpha_{ir}\right) \\ &\Pi_i' = \Pi_{i1}'\,[1 - L(\phi_i - \alpha_{i1})] + \sum_{j=2}^{m} \Pi_{ij}'\left[1 - L\left(\phi_i - \sum_{r=1}^{j}\alpha_{ir}\right)\right]\cdot L\left(\phi_i - \sum_{r=1}^{j-1}\alpha_{ir}\right) \\ &\Pi_i'' = \Pi_{i1}''\,[1 - L(\phi_i - \alpha_{i1})] + \sum_{j=2}^{m} \Pi_{ij}''\left[1 - L\left(\phi_i - \sum_{r=1}^{j}\alpha_{ir}\right)\right]\cdot L\left(\phi_i - \sum_{r=1}^{j-1}\alpha_{ir}\right) \end{aligned}\right\} \tag{6}$$

where $i = 1, \ldots, n$, $L(x)$ is a step function of the form

$$L(x) = \begin{cases} 0, & x < 0, \\ 1, & x \geq 0 \end{cases}.$$

Π_{ij}, Π_{ij}', Π_{ij}'' are the functions of position, the first transfer function, the second transfer function on parts of phase angles of actuation α_{ij} of mechanisms.

Equation (6) establish a connection between the Eq. (5) describe the dynamics of the automatic machine and the Eqs. (2)–(4) timing diagram of the machine-automaton. This

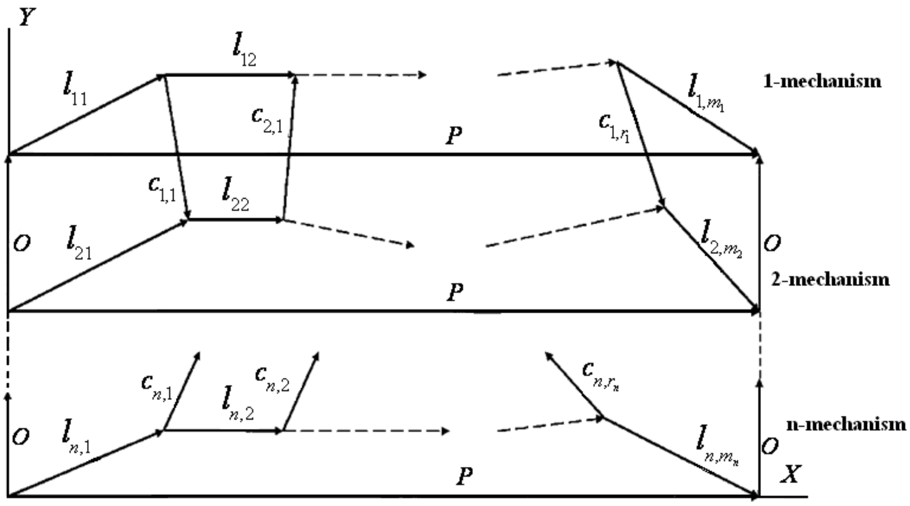

Figure 3. Vector model of the timing diagram of automatic machine.

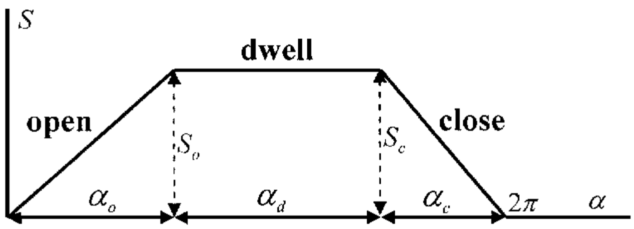

Figure 4. Diagram of the displacement of mechanism.

method allows to solve various optimization tasks, where the variable parameters are the phase angles α_{ij} and of the displacements δ_{ij} of timing diagram automatic machine.

4 An example

Let us show in more detail the connection between timing diagram of automatic machine and dynamics of mechanisms on the example of automatic machine with two cam mechanisms. The dynamic model is shown in Fig. 6, where φ_0, φ_1, φ_2 are generalized coordinates, M_D is the motor torque, M_1, M_2 are the moments of resistance, I_1, J_0, J_1, J_2 are the moments of inertia of mechanisms, c_0, c_1 are the coefficients of elasticity of shafts, $\Pi_i(\phi_i)$ is the function of position of cam mechanisms, $\Pi_i'(\phi_i)$ is the first transfer functions of the cams, $\Pi_i''(\phi_i)$ is the second transfer functions of the cams.

This dynamic model is described by the following equations

$$\left.\begin{array}{l} J_0\ddot{\phi}_0 + c_1(\phi_0 - \phi_1) = M_D, \\ \left(J_1 + I_1\Pi_1^{2'}(\phi_1)\right)\ddot{\phi}_1 + I_1\Pi_1'(\phi_1)\Pi_1''(\phi_1)\dot{\phi}_1^2 + c_0(\phi_1 - \phi_0) + c_1(\phi_1 - phi_2) = -M_1\Pi_1'(\phi_1), \\ \left(J_2 + I_2\Pi_2^{2'}(\phi_2)\right)\ddot{\phi}_2 + I_2\Pi_2'(\phi_2)\Pi_2''(\phi_2)\dot{\phi}_2^2 + c_2(\phi_2 - \phi_2) = -M_2\Pi_{2'}(\phi_2) \end{array}\right\} \quad (7)$$

where

$$\Pi_i'(\phi_i) = \frac{d\,\Pi_i(\phi_i)}{d\,\phi_i};\ \Pi_i''(\phi_i) = \frac{d^2\,\Pi_i(\phi_i)}{d\,\phi_i^2};\ i = 1,\ 2.$$

Figure 7 shows vector timing diagram of automatic machine, which is described by the following equations:

$$\left.\begin{array}{l} \boldsymbol{l}_{11} + \boldsymbol{l}_{12} = \boldsymbol{P} \\ \boldsymbol{l}_{21} + \boldsymbol{l}_{22} = \boldsymbol{P} \\ \boldsymbol{c}_{21} = \boldsymbol{l}_{21} - \boldsymbol{l}_{11} \end{array}\right\}. \quad (8)$$

Let us projected Eq. (8) on the x, y respectively

$$\left.\begin{array}{l} \alpha_{11} + \alpha_{12} = 2\pi \\ \alpha_{21} + \alpha_{22} = 2\pi \\ c_{11}^x = \alpha_{21} - \alpha_{11} \end{array}\right\} \quad (9)$$

$$\left.\begin{array}{l} \delta_{11} - \delta_{12} = 0 \\ \delta_{21} - \delta_{22} = 0 \\ c_{11}^y = \delta_{21} - \delta_{11} \end{array}\right\}. \quad (10)$$

Impose the constraints on the phase angles, displacements of mechanisms, and projections of vectors of connection

$$\left.\begin{array}{l} \alpha_{ij} \geq \alpha_{ij}^{\min} \\ \delta_{ij}^{\max} \geq \delta_{ij} \geq \delta_{ij}^{\min} \\ c_{11}^{x\max} \geq c_{11}^x \geq c_{11}^{x\min} \\ c_{11}^{y\max} \geq c_{11}^y \geq c_{11}^{y\min} \end{array}\right\}. \quad (11)$$

The expressions (Eqs. 9–11) allow you to vary the phase angles and displacements of mechanisms of automatic machine, without disrupting their normal work.

To establish the connection between the equations describing of joint work of the mechanisms of automatic machine (Eqs. 9–11) and the dynamic Eq. (7), let us write the functions of position and the transfer functions of mechanisms of automatic machine (see Fig. 8) as follows:

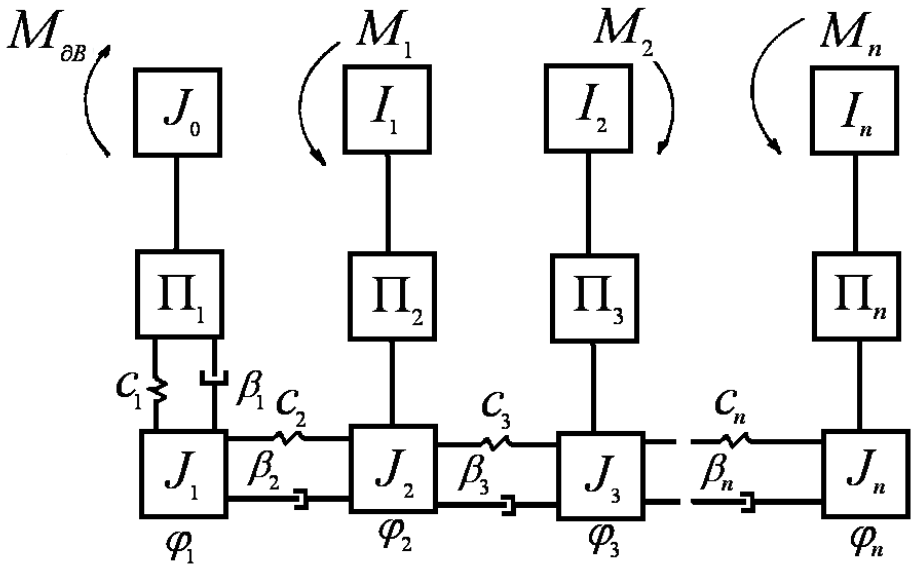

Figure 5. A dynamic model of automatic machine.

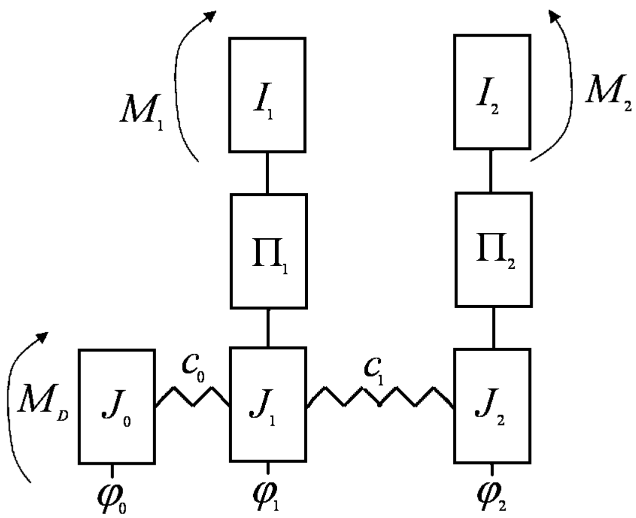

Figure 6. Dynamic model of automatic machine with two cams mechanisms.

$$\left.\begin{array}{l} \Pi_i = \Pi_{i1} \cdot [1 - L(\phi_i - \alpha_{i1})] + \Pi_{i2}\,[1 - L(\phi_i - (\alpha_{i1} + \alpha_{i2}))] \cdot L(\phi_i - \alpha_{i1}) \\ \Pi'_i = \Pi'_{i1} \cdot [1 - L(\phi_i - \alpha_{i1})] + \Pi'_{i2}\,[1 - L(\phi_i - (\alpha_{i1} + \alpha_{i2}))] \cdot L(\phi_i - \alpha_{i1}) \\ \Pi''_i = \Pi''_{i1} \cdot [1 - L(\phi_i - \alpha_{i1})] + \Pi''_{i2}\,[1 - L(\phi_i - (\alpha_{i1} + \alpha_{i2}))] \cdot L(\phi_i - \alpha_{i1}) \\ i = 1,\ 2 \end{array}\right\} \quad (12)$$

where

$$L(x) = \begin{cases} 0, & x < 0, \\ 1, & x \geq 0. \end{cases}.$$

Represent the generalized coordinates φ_1, φ_2 through the dimensionless coefficients k_1, k_2 where $\varphi_1 = 2\pi k_1$; $\varphi_2 = 2\pi k_2$; $k_1, k_2 \in [0, 1]$

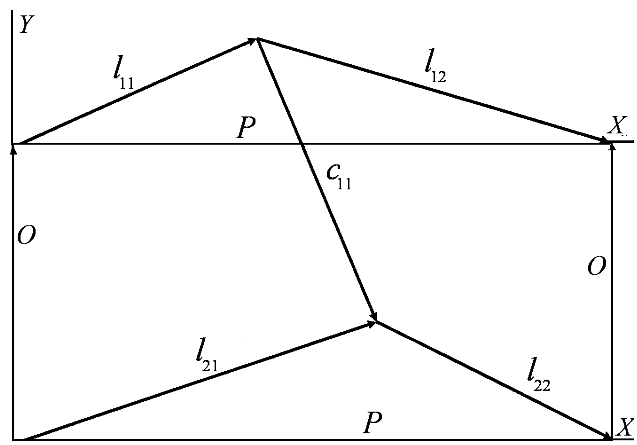

Figure 7. Vector timing diagram of automatic machine with two cams mechanisms.

$$\left.\begin{array}{l} \Pi_{ij} = a_{ij}(k_i)\,\delta_{ij} \\ \Pi'_{ij} = b_{ij}(k_i)\,\frac{\delta_{ij}}{\alpha_{ij}} \\ \Pi''_{ij} = d_{ij}(k_i)\,\frac{\delta_{ij}}{\alpha^2_{ij}} \\ i = 1,\ 2;\ j = 1,\ 2 \end{array}\right\}. \quad (13)$$

$a_{ij}(k_i)$, $b_{ij}(k_i)$, $d_{ij}(k_i)$; $i = 1, 2$; $j = 1, 2$; are coefficients of the displacement, the velocity, the acceleration of mechanism in j-position.

Equations (12) and (13) establish a connection between the phase angles of actuation of mechanisms α_{ij} and magnitude of displacement of mechanisms δ_{ij} and of their functions of position and transfer functions that are explicitly included in the equations of motion of the automatic machine (Eq. 7). Now, depending on the chosen optimization criterion, by

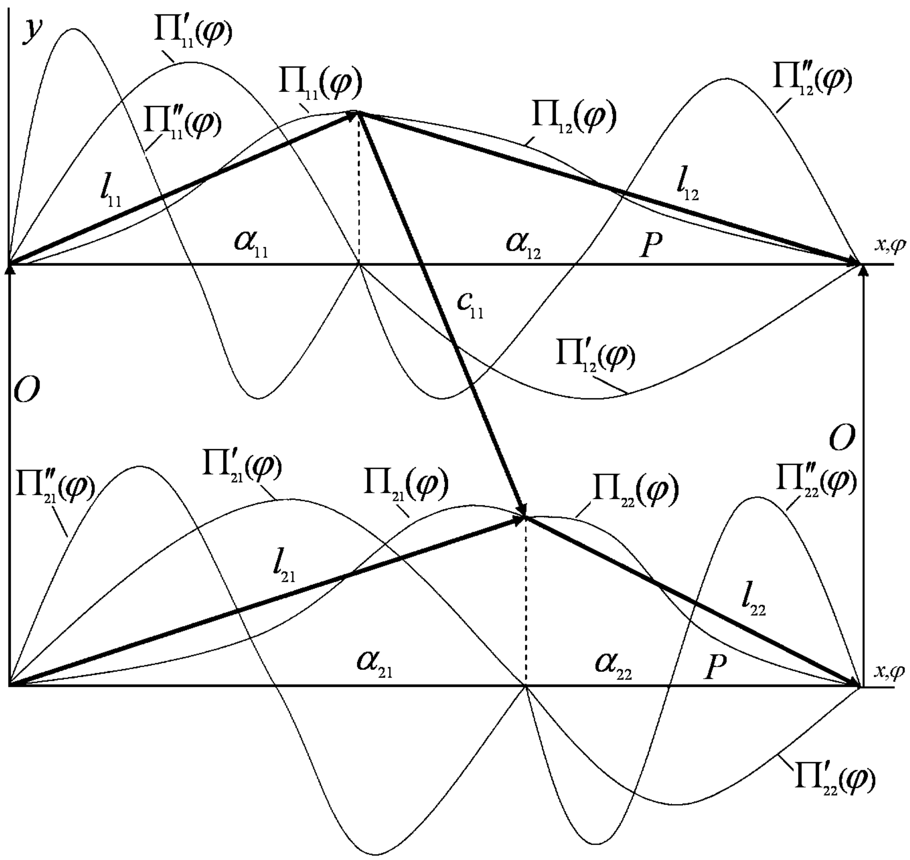

Figure 8. The functions of position and the transfer functions of mechanisms of automatic machine.

varying the parameters of timing diagram α_{ij} and δ_{ij}, can improve the dynamics of the automatic machine. As an optimization criterion can use the following expression:

$$\max_{\varphi_i}\left(\Pi'_i(\varphi_i)\,\Pi''_i(\varphi_i)\right).$$

5 Conclusions

The vector model of timing diagram is developed on the basis of representation timing diagram of the automatic machine as vector polygons, which allows solving various dynamic tasks at the expense of change of parameters of timing diagram.

The mathematical model of the automatic machine with elastic links on the basis of the timing diagram of its mechanisms is received.

The equations of connection between parameters timing diagram of the automatic machine and equations of dynamics through functions of position and transfer functions of mechanisms were received.

The model of timing diagram of the automatic machine is the only method which allows to solve a variety of dynamic tasks, by optimizing its timing diagrams, at this time.

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