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# Synthesis Theory and Optimum Design of Four-bar Linkage with Given Angle Parameters

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**Abstract.** In this paper, a synthesis method is proposed for the 5-point-contact four-bar linkage that approximates a straight line with given angle parameters. The given parameters were the angles and the location of the Ball point. Synthesis equations were derived for a general Ball–Burmester point case, the Ball–Burmester point at an inflection pole, and the Ball point that coincided with two Burmester points, resulting in three respective groups of bar linkages. Next, taking Ball–Burmester point as the coupler point, two out of the three bar-linkage combinations were used to generate three four-bar mechanisms that shared the same portion of a rectilinear trajectory. Computation examples were presented, and nine cognate straight-line mechanisms were obtained based on the Roberts-Chebyshev theory. Considering that the given parameters were angles which was arbitrarily chosen, with the other two serving as the horizontal and vertical axes, so the solution region graphs of the solutions for three mechanism configurations were plotted. Based on these graphs, the distribution of the mechanism attributes was obtained with high efficiency. By imposing constraints, the optimum mechanism solution was straightforwardly identified by the designers. For the angular parameters prescribed in this paper, the solutions for three straight-line mechanism configurations were obtained, along with nine cognate straight-line mechanisms that shared the same portion of the rectilinear trajectory. All the fixed pivot installation locations and motion performances differed, thus providing multiple solutions to the trajectory of the synthesis of mechanisms.

## 1 Introduction

The synthesis and optimization of mechanisms is a key technology in modern equipment innovations such as those in ship building, power locomotives and construction machinery, to name a few. As modern machinery continues to move toward greater automation and intelligence, due to the advantages of reliable support, strong bearing capacity, and easy processing, linkage mechanisms play an increasingly important role. As such, research on new synthesis methods and application technologies is attracting the attention of more and more specialists in the field of mechanics (Han, 1993; McCarthy, 2000). Brake et al. discussed the Complete Solution of Alt–Burmester Synthesis Problems for Four-Bar Linkages (Brake et al., 2016). Bulatović et al. (2016) developed a variable controlled deviations method and modified Krill Herd (MKH) algorithm to synthesize four-bar linkages for accomplishing approximately rectilinear motion (Bulatović and Dordević, 2009; Bulatović et al., 2016). Singh et al. (2017) used nature inspired optimization algorithms to reduce the computation and get the crank-rocker mechanisms without defects (Singh et al., 2017). Sleesongsoma and Bureerat (2017) proposed a variant of teachinglearning-based-optimization for four-bar linkage path generation, which was significantly superior to its original version (Sleesongsoma and Bureerat, 2017). Deshpande and Purwar (2017) proposed a novel algorithm for optimal approximate synthesis of

Burmester problem with no exact solutions (Deshpande and Purwar, 2017). Wang et al. (2019) developed a program package based on Matlab for the synthesis calculation of planar 4R linkage based on the theory of planar analytic geometry (Wang et al., 2019). Ramanpreet et al. proposed a refinement scheme for the optimal syntheses of the planar crank-rocker linkage free from all defects, which is used in human knee exoskeleton (Singh et al., 2017). Bulatović and Dordević (2009) proposed the variable controlled deviations method to synthesize planar four-bar mechanisms for accomplishing approximately rectilinear motion. Sleesongsoma and Bureerat (2017) introduced a variant of teachinglearning-based-optimization, which was significantly superior to its original version. Singh et al. (2017) proposed an optimization algorithm based on TLBO, which could reduce the computation and get the crank-rocker mechanisms without defects. A straight-line motion mechanism refers to one whose points occupy a portion of a trajectory that is approximately or precisely rectilinear (Vidosic and Tesar, 1967; Dijksman, 1976; Yu et al., 2013; Yin et al., 2019). Numerous researchers have worked on the synthesis theory and developed optimization methods for such mechanisms (Han et al., 2009; Han and Cao, 2018; Yang et al., 2011; Cui and Han, 2016). Chen et al. (2013, 2016) focused on the design and analysis of compliant Sarrus straight-line mechanisms, and developed several straight-line mechanisms with special performance (Chen et al., 2013, 2016).

In the practical application of hinged four-bar straight-line mechanisms, the designers usually have specific requirements regarding the installation locations, dimensions, and performance of the fixed pivots, and there can be an infinite number of mechanisms that might satisfy these requirements. Therefore, selection of the optimum mechanism solution that best satisfies the practical engineering conditions is a difficult problem that has puzzled designers.

In this paper, a synthesis method is proposed for the 5-point-contact four-bar linkage that approximates a straight line with given angle parameters. The given parameters were the angles and the location of the Ball point. Synthesis equations were derived for a general Ball–Burmester point case, the Ball–Burmester point at an inflection pole, and the Ball point that coincided with two Burmester points, resulting in three respective groups of bar linkages. Next, taking Ball–Burmester point as the coupler point, two out of the three bar-linkage combinations were used to generate three four-bar mechanisms that shared the same portion of a rectilinear trajectory. Computation examples were presented, and nine cognate straight-line mechanisms were obtained based on the Roberts-Chebyshev theory. Considering that the given parameters were angles which was arbitrarily chosen, with the other two serving as the horizontal and vertical axes, so the solution region graphs of the solutions for three mechanism configurations were plotted. Based on these graphs, the distribution of the mechanism attributes was obtained with high efficiency. By imposing constraints, such as the mechanism type, the ratio of the longest to the shortest link, the minimum transmission angle, and the length of approximate straight-line, the optimum mechanism solution was straightforwardly identified by the designers. For the angular parameters prescribed in this paper, the solutions for three straight-line mechanism configurations were obtained, along with nine cognate straight-line mechanisms that shared the same portion of the rectilinear trajectory. All the fixed pivot installation locations and motion performances differed, thus providing multiple solutions to the trajectory of the synthesis of mechanisms. The designers obtained intuitionally mechanism properties involved and avoided aimlessness in traditional optimum design methods mentioned in the references. The optimal mechanism with expected parameters could be selected more precisely and rapidly as the synthesizing process was visible and automatic.

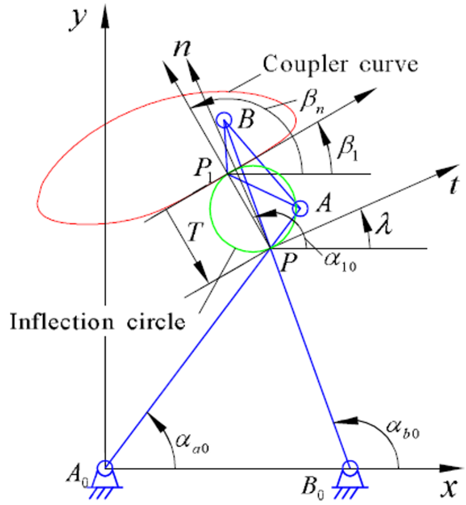


**Figure 1.** Definitions of various parameters (Yin et al., 2019).

## 2 Synthesis equations

Based on theories of kinematic geometry for points with infinite proximity, it is well known that the motion of a rigid body can be described as the pure rolling of its instantaneous center line on its fixed centrode. The curvature relationship of trajectory of any point on a moving system is determined in terms of Euler-Savary equation. The Euler-Savary equation is $\frac{1}{\boldsymbol{PA}}-\frac{1}{\boldsymbol{PA}_0}=\frac{1}{D\sin\alpha}$ or $\frac{1}{r}-\frac{1}{r+\rho}=\frac{1}{D\sin\alpha}$ or $\rho=\frac{r}{D\sin\alpha-r}$; Where, $D$ is defined as the diameter of inflexion circle; $\boldsymbol{PA}$ and $\boldsymbol{PA}_0$ are vectors, as shown in Fig. 1. The relevant fundamental theories and parameter definitions are described in detail in Yin et al. (2012) and Yin and Han (2011) and are not repeated here. Taking the instantaneous center pole $P$ as the coordinate origin, we draw a unit circle along the positive $y$ axis and tangent $x$ axis at the origin. The angles between $\boldsymbol{PA}$, $\boldsymbol{PB}$, $\boldsymbol{PC}$, $\boldsymbol{PP}_1$ and the positive $x$ axis are $\alpha_a$, $\alpha_b$, $\alpha_c$, and $\alpha_1$, respectively, as illustrated in Fig. 1.

Taking the second-order derivative of the Euler-Savary equation $\rho_m = \frac{r}{D\sin\alpha - r}$, we obtain:

$$\tan^4\alpha + \frac{N(M-2)}{M}\tan^3\alpha + \left(\frac{\mathrm{d}N}{\mathrm{d}\sigma} - 1\right)\tan^2\alpha + \frac{N^2\frac{\mathrm{d}M}{\mathrm{d}\sigma} - 3NM}{M^2}\tan\alpha + \frac{N^2(1-M)}{M^2} = 0, \quad (1)$$

where $M$ and $N$ are auxiliary variables: $\frac{1}{M} = \frac{1}{3}\left[\frac{1}{D} + \frac{1}{\rho_m}\right]$, $\frac{1}{N} = -\frac{1}{3D}\frac{\mathrm{d}D}{\mathrm{d}\sigma}$.

Equation (1) is a quartic equation for a single variable and it has four real roots at most. These four roots give the $\alpha_i$ of Burmester points' polar coordinates by employing Mueller's concepts on the highest attainable order of straight lines (Kwun-Lon Ting, 1991). Movable joints $A$, $B$, and $C$ correspond to the three roots of $\alpha_a$, $\alpha_b$, and $\alpha_c$, respectively. The coupler point $P_1$ corresponds to one root of $\alpha_1$.

From the relationship between the roots and coefficients of a quadratic equation, one obtains:

$$\tan^2\alpha + \left[\tan\alpha_a + \tan\alpha_b + \frac{N(M-2)}{M}\right]\tan\alpha + \frac{N^2(1-M)}{M^2\tan\alpha_a\tan\alpha_b} = 0, \quad (2)$$

where $\tan\alpha_1$ and $\tan\alpha_c$ are the two roots of the quadratic equation.

To simplify the calculation, in this paper, the diameter of the inflection circle is taken as $D = 1$. The final solution can be multiplied by the diameter of the practical inflection circle. Now, we solve for the joint coordinates of the four-bar mechanisms under various given conditions.

## 2.1 General case of Ball–Burmester point

For a general case of a Ball–Burmester point, the given parameters are the angles $\alpha_a$, $\alpha_b$, and $\alpha_1$. From Eq. (2), the value of $\alpha_c$ can be computed as follows:

$$\alpha_c = \tan^{-1}\left(-\frac{2\tan\alpha_1 + V}{U+1}\right), \quad (3)$$

where $U = \tan\alpha_a\tan\alpha_b$ and $V = \tan\alpha_a + \tan\alpha_b$.

By definition, one obtains the following:

$$\boldsymbol{PP}_1 = D\sin\alpha_1. \quad (4)$$

Therefore, the coordinates of the Ball–Burmester point can be obtained as follows:

$$\begin{cases} P_{1x} = \boldsymbol{PP}_1 \times \cos\alpha_1 \\ P_{1y} = \boldsymbol{PP}_1 \times \sin\alpha_1 \end{cases} \quad (5)$$

$\boldsymbol{PA}$, $\boldsymbol{PB}$ and $\boldsymbol{PC}$ can be computed from the following respective equations:

$$\boldsymbol{PA} = \frac{[(3U+1)\tan\alpha_1 + UV]\sin\alpha_a}{(U+1)\tan\alpha_a + U(2\tan\alpha_1 + V)}, \quad (6)$$

$$\boldsymbol{PB} = \frac{[(3U+1)\tan\alpha_1 + UV]\sin\alpha_b}{(U+1)\tan\alpha_b + U(2\tan\alpha_1 + V)}, \quad (7)$$

$$\boldsymbol{PC} = \frac{[(3U+1)\tan\alpha_1 + UV]\sin\alpha_c}{(U-1)(2\tan\alpha_1 + V)}. \quad (8)$$

From the geometric relationships, the coordinates of movable joints $A$, $B$, and $C$ are as follows:

$$\begin{cases} A_x = \boldsymbol{PA} \times \cos\alpha_a \\ A_y = \boldsymbol{PA} \times \sin\alpha_a \end{cases}, \quad (9)$$

$$\begin{cases} B_x = \boldsymbol{PB} \times \cos\alpha_b \\ B_y = \boldsymbol{PB} \times \sin\alpha_b \end{cases}, \quad (10)$$

$$\begin{cases} C_x = \boldsymbol{PC} \times \cos\alpha_c \\ C_y = \boldsymbol{PC} \times \sin\alpha_c \end{cases}. \quad (11)$$

From the equations:

$$\begin{cases} \boldsymbol{PA}_0 = -\frac{\boldsymbol{PA}\cdot D\sin\alpha_a}{\boldsymbol{PA} - D\sin\alpha_a} \\ \boldsymbol{PB}_0 = -\frac{\boldsymbol{PB}\cdot D\sin\alpha_b}{\boldsymbol{PB} - D\sin\alpha_b} \\ \boldsymbol{PC}_0 = -\frac{\boldsymbol{PC}\sin\alpha_c}{\boldsymbol{PC} - \sin\alpha_c} \end{cases}, \quad (12)$$

$\boldsymbol{PA}_0$, $\boldsymbol{PB}_0$, $\boldsymbol{PC}_0$ can be solved. In turn, the coordinates of fixed joints $A_0$, $B_0$, $C_0$ can be obtained as follows:

$$\begin{cases} A_{0x} = \boldsymbol{PA}_0 \times \cos\alpha_a \\ A_{0y} = \boldsymbol{PA}_0 \times \sin\alpha_a \end{cases}, \quad (13)$$

$$\begin{cases} B_{0x} = \boldsymbol{PB}_0 \times \cos\alpha_b \\ B_{0y} = \boldsymbol{PB}_0 \times \sin\alpha_b \end{cases}, \quad (14)$$

$$\begin{cases} C_{0x} = \boldsymbol{PC}_0 \times \cos\alpha_c \\ C_{0y} = \boldsymbol{PC}_0 \times \sin\alpha_c \end{cases}. \quad (15)$$

When the coordinates of all the joints are available, the three groups of bar linkages $AA_0$, $BB_0$, $CC_0$ can be obtained. Taking Ball–Burmester point $P_1$ as the coupler point, two out of the three bar linkage combinations can be used to generate three four-bar straight-line mechanisms.

## 2.2 Ball–Burmester point lying on the inflection pole

When the Ball–Burmester point is on inflection-circle pole, the given parameters are angle $\alpha_a$ and parameter $D'$, where $D'$ is the diameter when the trajectory of the circle center degenerates into a circle. Coupler point $P_1$ is the inflection pole. The remaining parameters of the mechanism are computed as follows:

$$\boldsymbol{PA}_0 = D'\sin\alpha_a, \quad (16)$$

$$\boldsymbol{PA} = \frac{\boldsymbol{PA}_0}{D'+1}, \quad (17)$$

$$\alpha_b = \tan^{-1}\left(-\frac{D'+1}{D'+3}\cdot\frac{1}{\tan\alpha_a}\right), \quad (18)$$

$$\boldsymbol{PB}_0 = D'\sin\alpha_b, \quad (19)$$

$$\boldsymbol{PB} = \frac{PB_0}{D'+1}. \quad (20)$$

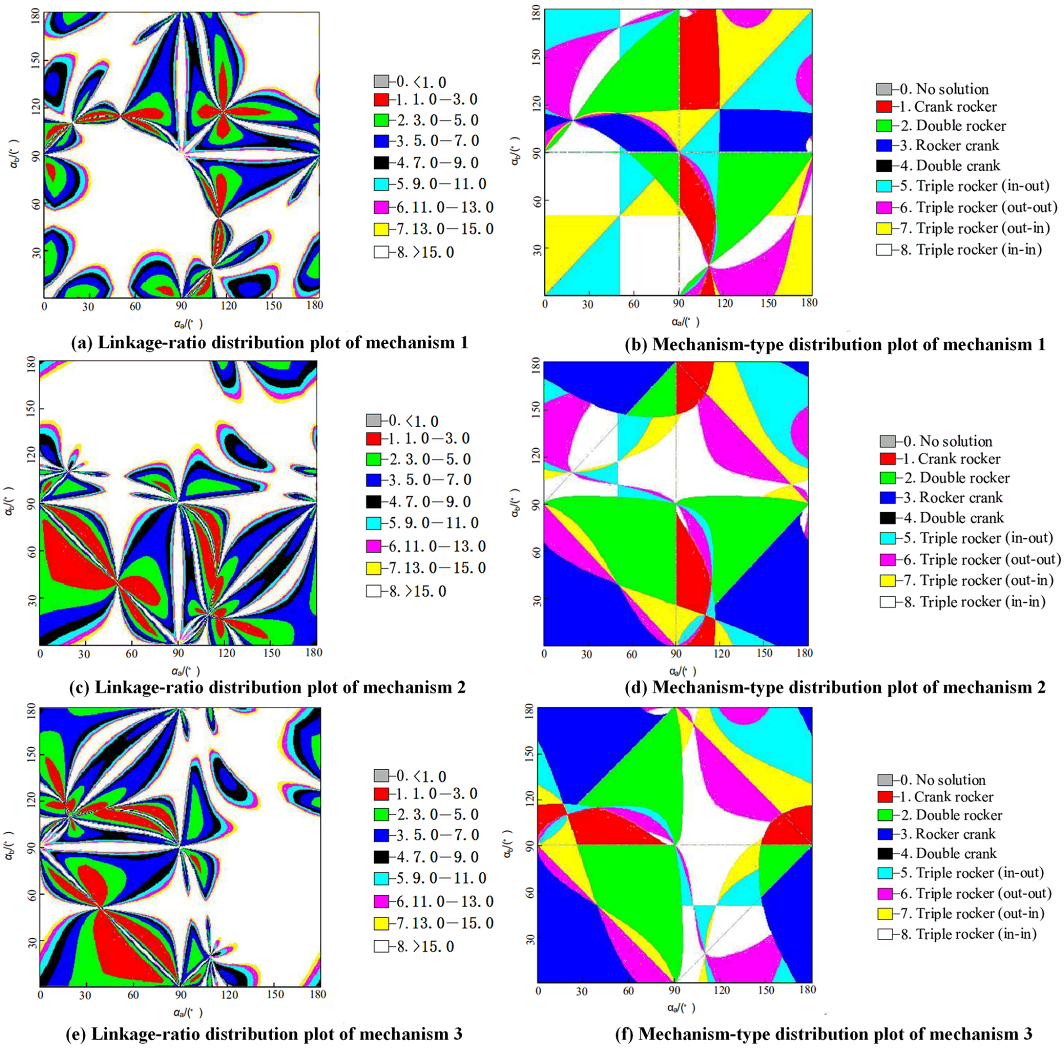


**Figure 2.** Linkage ratio distribution and Mechanism type distribution plots.

**Table 1.** Parameters of mechanisms.

| Mechanisms | The length of links | | | | | $l_{sum}$ | $\frac{l_{max}}{l_{min}}$ | Type of mechanisms |
|---|---|---|---|---|---|---|---|---|
| | $A_0A$ | $AB$ | $B_0B_0$ | $A_0B_0$ | $AP_1$ | | | |
| $M_{1-1}$ | 258.18 | 89.66 | 85.56 | 100.56 | 73.10 | 533.96 | 3.01 | 5-three rocker |
| $M_{1-2}$ | 258.18 | 185.08 | 31.14 | 106.23 | 73.10 | 580.63 | 8.29 | 3-rocker-crank |
| $M_{1-3}$ | 85.56 | 95.42 | 31.14 | 48.44 | 82.63 | 260.56 | 3.06 | 3-rocker-crank |
| $M_{2-1}$ | 45.60 | 65.39 | 114 | 196.16 | 94.66 | 421.15 | 4.30 | 8-three rocker |
| $M_{2-2}$ | 45.60 | 27.48 | 8.33 | 65.04 | 94.66 | 146.45 | 7.81 | 3-rocker-crank |
| $M_{2-3}$ | 114 | 37.91 | 8.33 | 143.54 | 59.18 | 303.78 | 17.23 | 3-rocker-crank |
| $M_3$ | 8.48 | 23.39 | 42.48 | 56.61 | 59.90 | 130.96 | 6.68 | 1-crank-rocker |

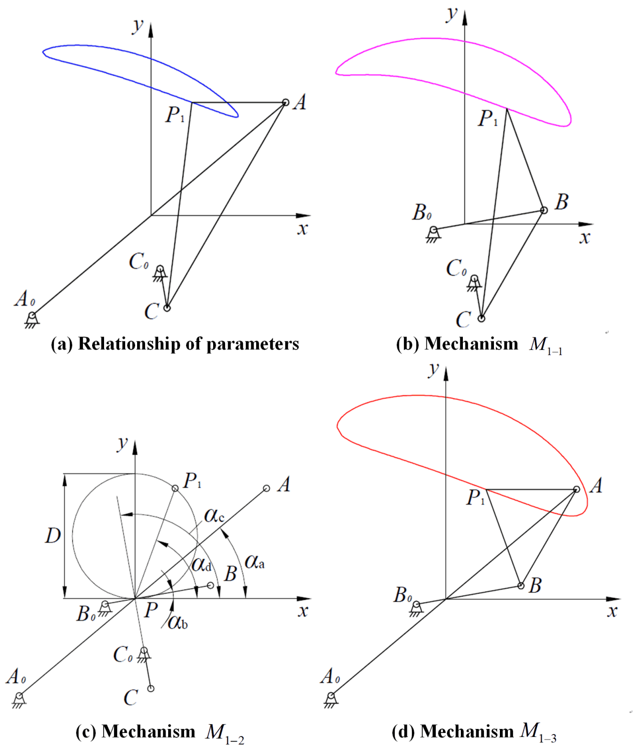


**Figure 3.** Mechanism plots.

In this situation, $\alpha_c = 90°$.

$$\boldsymbol{PC} = \frac{D'}{2D'+4} \tag{21}$$

$$\boldsymbol{PC}_0 = \frac{D'}{D'+4} \tag{22}$$

After computing the above parameters, similar to Eq. (1), the coordinates of movable joints $A$, $B$, and $C$ and fixed joints $A_0$, $B_0$, and $C_0$ can be solved to obtain three four-bar straight-line mechanism.

## 2.3 Ball point coinciding with two Burmester points

When two Burmester points coincide with the Ball point, the given parameters are angles $\alpha_a$ and $\alpha_b$. Every group of given parameters can only generate one four-bar straight-line mechanism. The remaining parameters of this mechanism are computed as follows:

$$\alpha_1 = \tan^{-1}\left(-\frac{V}{U+3}\right), \tag{23}$$

where $U = \tan\alpha_a \tan\alpha_b$ and $V = \tan\alpha_a + \tan\alpha_b$.

By substituting Eq. (23) into $\boldsymbol{PP}_1 = D\sin\alpha_1$, $\boldsymbol{PP}_1$ can be solved.

$$\boldsymbol{PA} = \frac{V(U-1)\sin\alpha_a}{(U+3)\tan\alpha_a + U\cdot V} \tag{24}$$

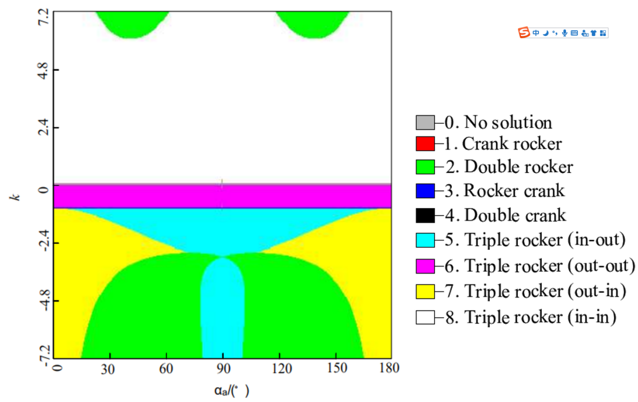


**Figure 4.** Mechanism type distribution plot.

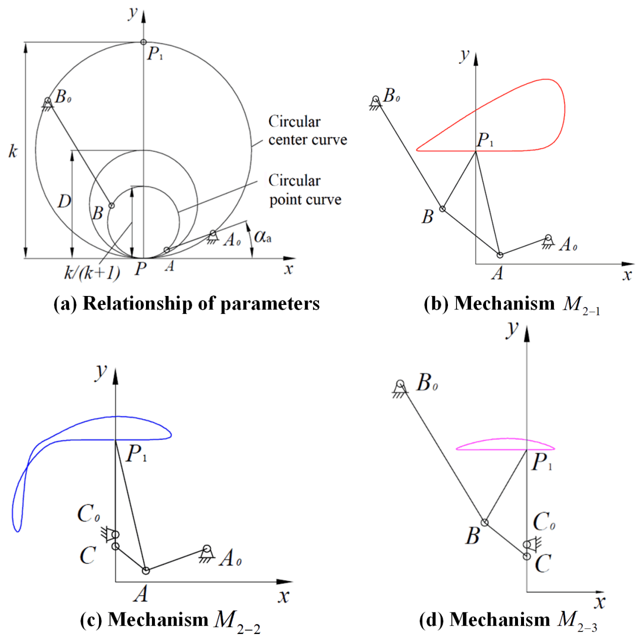


**Figure 5.** Mechanism plots.

$$\boldsymbol{PB} = \frac{V(U-1)\sin\alpha_b}{(U+3)\tan\alpha_b + U\cdot V} \tag{25}$$

By substituting Eqs. (24) and (25) into the following two equations:

$$\begin{cases} \boldsymbol{PA}_0 = -\dfrac{\boldsymbol{PA}\cdot D\sin\alpha_a}{\boldsymbol{PA}-D\sin\alpha_a} \\ \boldsymbol{PB}_0 = -\dfrac{\boldsymbol{PB}\cdot D\sin\alpha_b}{\boldsymbol{PB}-D\sin\alpha_b} \end{cases}, \tag{26}$$

$\boldsymbol{PA}_0$ and $\boldsymbol{PB}_0$ can be obtained. After the above parameters have been computed, the coordinates of movable joints $A$, $B$, and $C$ and fixed joints $A_0$, $B_0$, and $C_0$ can be solved to obtain one four-bar straight-line mechanism.

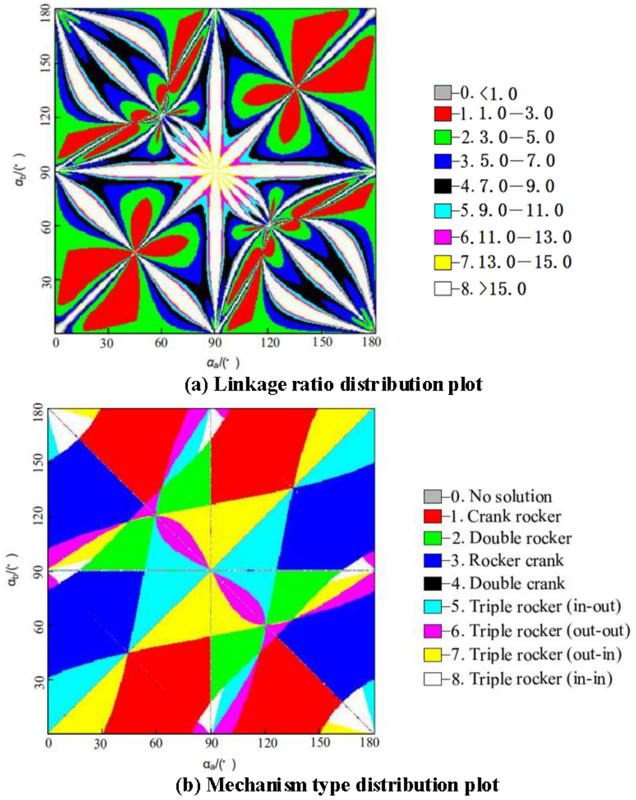


**Figure 6.** Linkage-ratio distribution and mechanism-type distribution plot.

## 3 Solution region analysis and synthesis examples

Now, we adopted the mechanism solution region method, taking the inflection circle diameter $D = 1$ and the mechanism-type distribution plot as an example to draw the mechanism solution region graphs for all three conditions.

### 3.1 General Case of the Ball–Burmester Point

Without losing generality, let $\alpha_1 = 70°$, take $\alpha_a$ and $\alpha_b$ as the horizontal and vertical axes, respectively, and let $\alpha_a$ and $\alpha_b$ take continuous values from 0 to 180° to obtain solution region graphs for three mechanism configurations, as illustrated in Fig. 2 (Barker, 1985).

By arbitrarily choosing $\alpha_a = 40°$ and $\alpha_b = 10°$ in Fig. 2, Fig. 3b shows the obtained mechanism, and Fig. 3c and d show the two corresponding mechanism solutions. Table 1 lists the mechanism parameters.

### 3.2 Ball–Burmester point lying on the inflection pole

Take $\alpha_a$ as the horizontal axis and the diameter of the degenerated circular center curve $D'$ as the vertical axis and let parameter $D'$ take continuous values from −7.2 to 7.2 and angle $\alpha_a$ take continuous values from 0 to 180°, then the solution region graphs of the three mechanism configurations

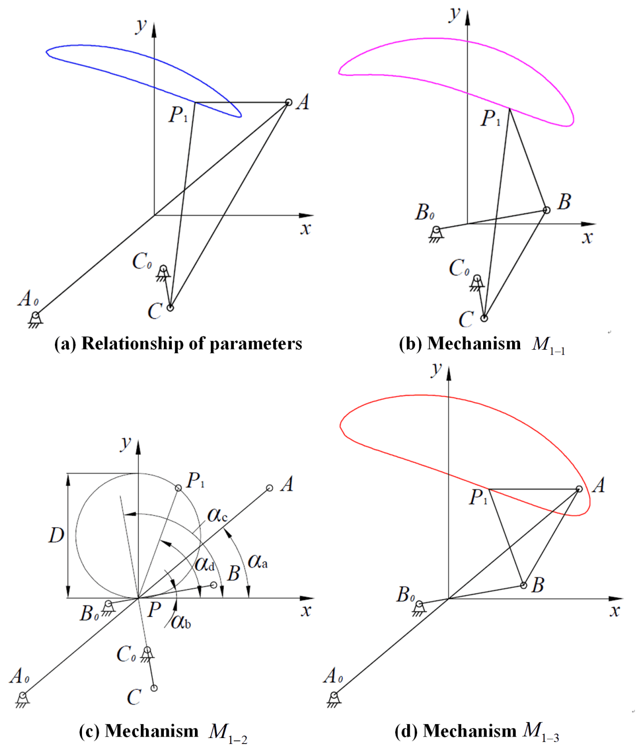


**Figure 7.** Mechanism $M_3$.

can be obtained. Figure 4 shows the mechanism-type distribution graph for the first configuration $A_0AB_0BP_1$.

By arbitrarily choosing $\alpha_a = 20°$ and $D' = 2$ in Fig. 4, Fig. 5b shows the obtained mechanism, and Fig. 5c and d show the other two mechanisms. Table 1 lists the mechanism parameters.

### 3.3 Ball point coinciding with two Burmester points

Similar to Eq. (1), by taking $\alpha_a$ as the horizontal axis and $\alpha_b$ as the vertical axis, we obtained the mechanism type of the single mechanism configuration $A_0AB_0BP_1$ and its linkage-ratio distribution plot, as shown in Fig. 6. By arbitrarily choosing $\alpha_a = 100°$ and $\alpha_b = 150°$ in Fig. 6, Fig. 7 shows the obtained mechanism. Table 1 shows the mechanism parameters.

## 4 Conclusion

Using the synthesis method proposed in this paper combined with the cognate mechanism theory, nine different four-bar mechanisms with identical rectilinear trajectory sections were obtained that have different frame locations and performances for the designer to choose. Given that the known parameters were angular, this method was used to obtain the solution region graphs of three mechanism solutions. Based on these solution region graphs, the distribution of the attributes of the mechanism solutions was obtained with high efficiency, and the optimum solution was extracted in a straightforward manner. The optimum design mentioned in the paper was to choose optimum mechanism from the infinite number of mechanism solutions. By imposing constraints, such as the mechanism type, the ratio of the longest to the shortest link, the minimum transmission angle, and the length of approximate straight-line, the optimum mechanism solution was straightforwardly identified by the designers. The design data have been obtained and converted into a series of design graphs by the computer program which can be used to synthesize easily four-bar linkages yielding desired straight-line outputs of predetermined position. The method proposed in this paper represents is a new approach to the synthesis of classic straight-line mechanisms and has high value in practical applications.

**Data availability.** All the data used in this manuscript can be obtained on request from the corresponding author.

**Author contributions.** LY proposed the idea and methodology; LH derived the equations; JH, PX, XP and PZ developed the software.

**Competing interests.** The authors declare that they have no conflict of interest.

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