Fig. 3: Singular shapes of the 3-PPR PKM. The shape of the base platform in (a) can be viewed as a straight line, while the base platforms in (b) and (c) are isosceles trapezoid and parallelogram, respectively, if we virtually close the opening by drawing a line parallel to the bottom line.

The system of coefficient equations yields the same results as Eq. (23).

4 Shape singularity of a 3-PPS PKM

The second example is about a spatial PKM, namely, a 3-PPS PKM. The model of general 3-PPS PKM is shown in Figs. 4(a) and 4(b). A special case of 3-PPS PKM discussed in the work of Bonev (2008) is depicted in Fig. 4(c).

The shape of MP is defined as an isosceles triangle with side length $h_1$ and vertex angle $2\alpha$, the feasible range of the angle being defined as $0^\circ \leq 2\alpha \leq 180^\circ$. The base platform is defined in a similar way with side length $h_2$ and vertex angle $2\beta$ ($0^\circ \leq 2\beta \leq 180^\circ$). The axes of the passive prismatic joints are perpendicular to the axes of the actuated prismatic joints, while the orientation of axes of the actuated prismatic joints is described by the pan angle $\delta$ ($0^\circ \leq \delta \leq 90^\circ$) and tilt angle $\gamma$ ($0^\circ \leq \gamma \leq 180^\circ$).

4.1 Kinematics of the 3-PPS PKM

We formulate first the kinematics model of mechanism by including shape parameters. As shown in Fig. 4(b), the global frame $\{O_0\}$ is attached to the base with its origin located at the centroid of the base, where $Y_0$-axis is pointed along the direction of $B_2B_3$, $Z_0$ is perpendicular to the base. Local frame $\{O\}$ is attached to the MP and defined in a similar way.

For general 3-PPS PKM ($\delta \neq 0^\circ, 90^\circ$), let $(x, y, z, \psi, \theta, \phi)$ describe the pose of the MP with respect to the global frame. The position vector of point $A_i$ in general 3-PPS PKM is obtained by

$$a_i = Ra'_i + p, i = 1, 2, 3$$

where $a'_i$ is the position vector of point $A_i$ in the local frame, $R = R_z(\phi)R_y(\theta)R_x(\psi)$ is the rotation matrix and $p = [x, y, z]^T$ is