\[ g = 12ahsac\beta(c\beta)^2 - 2a^2sacbcac\beta + 18h^2sacbcac\beta - 2a^2(cac\beta)^2c\beta^2 \\
+ 18h^2(cac\beta)^2(c\beta)^2 - 12ahsbc(c\beta)(c\alpha)^2 - 12ahsac\alpha \\
+ 3a^2(c\beta)^2 - 9h^2(c\beta)^2 + 12ahsbc\beta + 3a^2(c\alpha)^2 - 9h^2(c\alpha)^2 - 4a^2 = 0 \] (22)

This equation stands for the manifold of all shape parameters that leads to shape singularity. Figures 2(a) and 2(b) show the 3D surface of function \( g \), plotted for two given values of \( \theta \).

It can be seen from the plots of \( g \) that there are some points where \( g \) becomes zero, which are the points of shape singularity. Equation (22) was solved analytically by virtue of computer algebra, which yields three singular shapes

\[
\begin{align*}
(1) & : \alpha = 0^\circ \text{ or } 180^\circ, \quad \beta = 0^\circ \text{ or } 180^\circ, \quad \theta = \theta \\
(2) & : \alpha = \alpha, \quad \beta = \pi - \alpha, \quad \theta = \alpha - 90^\circ \\
(3) & : \alpha = \alpha, \quad \beta = \alpha, \quad \theta = 0^\circ
\end{align*}
\] (23)

The corresponding manipulators are shown in Figs. 3(a)-3(c). For the first singular shape, as shown in Fig. 3(a), the lines of translation of the active prismatic joints are parallel, and perpendicular to the lines of translation of the three passive prismatic joints. The MP can still move along the direction of the passive prismatic joints even though all the three active joints are locked.

For the second singular shape depicted in Fig. 3(b), the MP can perform a small rotation about an instantaneous center of rotation (\( G_1 \)), even all the actuators are locked. For the third singular shape, as shown in Fig. 3(c), a small rotation of the MP about an instantaneous center of rotation (\( G_2 \)) is feasible, even with all the actuators locked. The instantaneous rotation of the MP in both cases can also be easily interpreted by Arnold-Kennedy theorem (Mohammadi, 2005).

In the aforementioned three singular shapes, there are infinitive many designs associated with each shape. Take the second shape as example, the manipulator is singular while \( \alpha, \beta, \theta \) can take any values satisfying Eq. (23b), but the shapes of the base and MP remain the same. In other words, it is only the shape that determines this type of singularity.

In the above formulation, if we include \( l \) as a shape variable too, a trivial solution of \( l = 0 \) will be obtained, which means the MP becomes geometrically a point.

The formulation of Eq. (22) is derived to display the manifold and to obtain the solutions of shape singularity. If our interest is only the shape singularity solution, an alternative formulation is readily obtained by letting all coefficients equal to zero, namely,

\[
M_1 = 0; N_1 = 0; M_2 = 0; N_2 = 0; \quad (24)
\]